Exam 2 Review

Definitions: Be familiar with the following terms

Linear Transformations, Matrix Transformations, invertible matrix, non-singular matrix, determinant, cofactor expansion, eigenvectors, eigenvalues, eigenspace

Linear Transformations:

1) Consider the square with endpoints (0,0) (1,0), (1,1) (0,1) Find a linear transformation A such that the multiplication of A transforms the square into a parallelogram with end points (0,0) (4,2) (2,4), (6,6).

2) Prove that A= $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is a linear transformation.

Matrix Multiplication:

3) Let A= $\begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$, B= $\begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}$. With direct calculation show that $(AB)^T = B^T A^T$

4) Define a transformation T: $\Re^2 \to \Re^2$ by the following rule: T(X) is the result of first rotating X counter clockwise by 45 degrees and then multiplying by

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}.$$

Find a matrix B such that T(X)=BX for all $X \in \Re^2$

Inverses:

5) Without using a calculator find A^{-1} when $A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

6) List at least three statements that are equivalent to where A is a M(n,n).

a) A is invertible Matrix.
b)
c)
d)

7) Solve the following equation for X=[x1,x2,x3]^T

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 3 \\ 3 & 4 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Determinants:

8) Without using a calculator find det(A) where:

$$A = \begin{bmatrix} 2 & 0 & 2 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 3 & 2 \\ 1 & 0 & 0 & 5 \end{bmatrix}$$

9) Without using a calculator find det(A) where:

$$A = \begin{bmatrix} 7 & 1 & 1 \\ 0 & a & b \\ 0 & c & d \end{bmatrix}$$

9) Let $A = \begin{bmatrix} 5 & 3 \\ 1 & 3 \end{bmatrix}$

a) Find the eigenvalues of A
b) Find the eigenvectors of A.
c) Find a basis for the eigenspace of A
d) Calculate $A^{10}B$ where B=$[3,1]^T$