

Final Review:

Be prepared to answer all questions on the first two exams and the review for the 2nd exam. Also be prepared to answer the following types of questions:

1) Find the distance between $u = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ and $v = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$

2) Suppose that a vector y is orthogonal to vectors u and v . Show that y is orthogonal to the vector $u+v$

3) Determine if the following sets of vectors are orthogonal:

$$w = \left\{ \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -5 \\ -2 \\ 1 \end{bmatrix} \right\}$$

4) Show that $w = \left\{ \begin{bmatrix} 3 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 6 \end{bmatrix} \right\}$ is an orthogonal basis. Express $x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ as linear combination of the vectors in w .

5) Let $y = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $u = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$. Write y as a sum of a vector in the span $\{u\}$ and a vector orthogonal to u .

6) Find the best approximation to z by vectors of the form $c_1 v_1 + c_2 v_2$

$$z = \begin{bmatrix} 3 \\ -7 \\ 2 \\ 3 \end{bmatrix} \quad \text{and} \quad v_1 = \begin{bmatrix} 2 \\ -1 \\ -3 \\ 1 \end{bmatrix} \quad v_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}$$

7) Use the Gram-Schmidt process to produce an orthogonal basis for

$$w = \left\{ \begin{bmatrix} 0 \\ 4 \\ 2 \end{bmatrix}, \begin{bmatrix} 5 \\ 6 \\ -7 \end{bmatrix} \right\}$$

8) Find a least-squares solution of the following inconsistent system $Ax=b$ for

$$A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}$$