We reduce

$$\begin{bmatrix} 1.0 & 10,000.0 & 10,000 \\ 1.0 & 1.0 & 2.0 \end{bmatrix}$$

$$\begin{bmatrix} 1.0 & 10,000.0 & 10,000.0 \\ 0.0 & -9,999.0 & -9,998.0 \end{bmatrix}$$

yielding

$$y = 9998/9999 = 0.9998999900$$
$$x = 10,000 - 10,000y = 1.000100010001$$

Suppose that our computer only carries three places after the decimal. To three places, the answer is $y = 1.000$, $x = 1.000$. This, however, is not what our computer tells us! Our computer first rounds y to 1.000 and then computes $x = 10,000 - 10,000y = 0$, which is not even close.

The problem was caused by the division by 0.0001 in our first step. This can be avoided by interchanging the rows before doing the reduction:

$$\begin{bmatrix} 1.0 & 1.0 & 2.0 \\ 0.0001 & 1.0 & 1.0 \end{bmatrix}$$
$$\begin{bmatrix} 1.0 & 1.0 & 2.0 \\ 0 & 0.9999 & 0.9998 \end{bmatrix}$$

yielding $y = 1.000$ and $x = 2 - 1 \cdot 1 = 1.000$.

To avoid such problems, many computational algorithms rearrange the rows before each reduction step so that the entry in the pivot position is the one of largest absolute value. This process is called **partial pivoting**. (There is also a process called **full pivoting**, that involves rearranging both rows and columns so as to obtain the largest pivot entry.) Unfortunately, even full pivoting will not eliminate all round-off difficulties. There are certain systems, called **ill-conditioned**, that are inherently sensitive to small inaccuracies in the values of the coefficients on the right sides of the equations in the system. There is a number, the **conditioning number**, that measures this sensitivity. The conditioning number is discussed in the computer exercises for Section 3.3.

True-False Questions

Provide a proof for each true statement and a counter example for each false statement. In these questions, assume that R is the echelon form of the augmented matrix for a system of equations.

1. If the system has three unknowns and R has three nonzero rows, then the system has at least one solution.

2. If the system has three unknowns and R has three nonzero rows, then the system can have an infinite number of solutions.

3. The system below has an infinite number of solutions:

$$\begin{aligned} 2x + 3y + 5z + 6w - 7u - 8v &= 0 \\ 3x - 4y + 7z + 6w + 8u + 5v &= 0 \\ -7x + 9y - 2z - 4w - 5u + 2v &= 0 \\ -5x - 5y + 9z + 3w + 2u + 7v &= 0 \\ -9x + 3y - 9z + 5w - 3u - 4v &= 0 \end{aligned}$$

4. The following matrix may be reduced to reduced echelon form *using only one elementary row operation*:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

5. The matrix in question 4 is the coefficient matrix for a consistent system of equations.

Exercises

A check mark ✓ next to a problem number indicates that there is an answer or a hint provided in the Answers and Hints section at the end of the text.

In these exercises, if you are asked for a general solution, the answer should be expressed in "parametric form" as in the text. Indicate the spanning and translation vectors.

1. ✓Which of the following matrices are in echelon form? Which are in reduced echelon form?

(a)

$$\begin{bmatrix} 1 & 0 & 2 & 4 & 0 & 1 \\ 1 & 1 & 0 & 4 & 3 & 2 \\ 0 & 1 & 0 & 4 & 3 & 2 \\ 0 & 0 & 0 & 2 & 1 & 3 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 0 & 1 & 2 & 2 & 4 \\ 0 & 0 & 1 & 2 & 4 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(c)

$$\begin{bmatrix} 0 & 0 & 2 & 4 & 1 \\ 3 & 1 & 2 & 6 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & -1 & 1 \end{bmatrix}$$

2. Bring each of the matrices in Exercise 1 that are not already in echelon form to echelon form. Interpret each matrix as the augmented matrix for a system of equations. Give the system and general solution for each system.

3. Find the reduced echelon form of each of the

(a) ✓

$$\left[\begin{matrix} 2 & 7 & -5 \\ 1 & 0 & 1 \\ 1 & 3 & -2 \end{matrix}\right.$$

(b)

$$\left[\begin{matrix} 1 & 1 & 1 \\ 2 & 2 & 1 \\ 1 & 0 & 2 \\ 4 & 3 & 2 \end{matrix}\right.$$

(c) ✓

$$\left[\begin{matrix} 3 & 9 \\ 2 & 7 \end{matrix}\right.$$

(d)

$$\left[\begin{matrix} 2 & 1 & 3 \\ -2 & -1 & -3 \\ 4 & 2 & 7 \end{matrix}\right.$$

(e) ✓

$$\left[\begin{matrix} 5 & 4 \\ 1 & 2 \end{matrix}\right.$$

(f)

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix},$$ whe

(g) ✓

$$\left[\begin{matrix} 2 & 4 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{matrix}\right.$$

(h)

$$\left[\begin{matrix} 1 & 2 & 3 \\ 0 & 5 & 6 \\ 0 & 0 & 9 \\ 0 & 0 & 0 \end{matrix}\right.$$

(i)

$$\left[\begin{matrix} a & b & c \\ 0 & e & f \\ 0 & 0 & h \\ 0 & 0 & 0 \end{matrix}\right.$$

where a, e, h, and j are all nonzero.

(j)

$$\left[\begin{matrix} 2 & 5 \\ 1 & 4 \\ -1 & 2 \\ 2 & -1 \end{matrix}\right.$$

4. Suppose that the matrices in Exercise 3 are th of equations. In each case, write the system to the system.

3. Find the reduced echelon form of each of the following matrices:

(a) ✓

$$\begin{bmatrix} 2 & 7 & -5 & -3 & 13 \\ 1 & 0 & 1 & 4 & 3 \\ 1 & 3 & -2 & -2 & 6 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 1 & 1 & 1 \\ 1 & 0 & 2 & 3 & 2 \\ 4 & 3 & 2 & 1 & 0 \end{bmatrix}$$

(c) ✓

$$\begin{bmatrix} 3 & 9 & 13 \\ 2 & 7 & 9 \end{bmatrix}$$

(d)

$$\begin{bmatrix} 2 & 1 & 3 & 4 & 0 & -1 \\ -2 & -1 & -3 & -4 & 5 & 6 \\ 4 & 2 & 7 & 6 & 1 & -1 \end{bmatrix}$$

(e) ✓

$$\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$$

(f)

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}, \quad \text{where } ad - bc \neq 0$$

(g) ✓

$$\begin{bmatrix} 2 & 4 & 3 & 0 & 6 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 2 & 4 \end{bmatrix}$$

(h)

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 5 & 6 & 7 \\ 0 & 0 & 9 & 10 \\ 0 & 0 & 0 & 13 \end{bmatrix}$$

(i)

$$\begin{bmatrix} a & b & c & d \\ 0 & e & f & g \\ 0 & 0 & h & i \\ 0 & 0 & 0 & j \end{bmatrix}$$

where a, e, h, and j are all nonzero.

(j)

$$\begin{bmatrix} 2 & 5 & 11 & 6 \\ 1 & 4 & 9 & 5 \\ -1 & 2 & 5 & 3 \\ 2 & -1 & -3 & -2 \end{bmatrix}$$

4. Suppose that the matrices in Exercise 3 are the augmented matrices for a system of equations. In each case, write the system down and find all solutions (if any) to the system.

5. Prove the consistency of the solution to system (1.25) obtained in Section 1.2 [equation (1.22) on page 35] with the solution given in equation (1.31) on page 50 in this section.

(*Hint:* In deriving equation (1.31) we set $t = y$ and $s = w$. According to equation (1.22) $y = 1 + r + s$ and $w = s$. In equation (1.31), replace t by $1 + r' + s'$ and s by s' and simplify to obtain equation (1.22) with r and s replaced by r' and s'. What you have shown is that every vector X expressible in the form given in equation (1.31) is also expressible in the form given in equation (1.22). You must also prove that every vector X expressible in the form given in equation (1.22) is also expressible in the form given in equation (1.31).)

6. We proved that which variables are pivot variables and which are not does not depend on how we reduce the system. This is true only if we keep the variables in the same order.

(a) Find the pivot variables and the general solution for the following system:

$$\begin{aligned} 2x + 2y + 2z + \ \ 3w &= 4 \\ x + \ \ y + \ \ z + \ \ \ \ w &= 1 \\ 2x + 3y + 4z + \ \ 5w &= 2 \\ x + 3y + 5z + 11w &= 9 \end{aligned} \tag{1.36}$$

(b) Solve the equivalent system obtained by commuting the terms involving z and y in each equation. List the pivot variables and give the general solution. Express the general solution in the form $[x, y, z, w]^t = \ldots$.

(c) Prove the consistency of the expressions for the general solution obtained in parts (a) and (b) of this exercise. (*Hint:* Use reasoning similar to that suggested in the hint for Exercise 5.)

7. Repeat Exercises 6b and 6c for the system obtained from system (1.36) by commuting the terms involving z and w in each equation.

8. Show that commuting the terms involving x and y in each equation of system (1.36) does not change the pivot variables. How can we know, without any further work, that the expression for the solution to system (1.36) obtained by solving this new system will be identical with that obtained in part (a) of Exercise 6.

9. The following vectors are the translation vecto T and spanning vectors X_1 and X_2, obtained by using Gaussian elimination to solve a linear system of three equations in the variables x, y, z, and w.

$$T = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad X_1 = \begin{bmatrix} -3 \\ 4 \\ 1 \\ 0 \end{bmatrix}, \quad X_2 = \begin{bmatrix} 1 \\ 4 \\ 0 \\ 1 \end{bmatrix},$$

(a) Which are the nonpivot variables? H
equation (1.28), the general solution
Write out this expression using th
variable equals r and which variabl

(b) ✓ Find a row reduced echelon matr
system having the stated translation
there is only one such matrix. (*Hint:*
pivot variables. Write out the gener
matrix and choose the nonzero entri
translation and spanning vectors for

10. Redo Exercise 9, using

$$T = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad X_1 = \left[\begin{matrix} a \\ b \\ 1 \\ 0 \end{matrix} \right.$$

11. Redo Exercise 9 using the variables $x_1, \ldots$

$$T = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad X_1 = \begin{bmatrix} a \\ b \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad X_2$$

12. The following vectors T and X_1 are, respe
vectors for a system of four linear equatior
augmented matrix for the system. (*Hint:*
variables. Then write the most general R
having these variables as pivots. Compare
for the general system with those given.)

(a) ✓

$$T = \begin{bmatrix} -7 \\ 0 \\ 3 \\ 0 \end{bmatrix}, \quad X_1 =$$

(b)

$$T = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad X_1 = \Big[$$

(c)

$$T = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad X_1 =$$

(a) Which are the nonpivot variables? How can you tell? (*Hint:* According to equation (1.28), the general solution to this system is $X = T + rX_1 + sX_2$. Write out this expression using the given vectors to determine which variable equals r and which variable equals s.)

(b) ✓Find a row reduced echelon matrix that is the augmented matrix for a system having the stated translation and spanning vectors. Explain why there is only one such matrix. (*Hint:* From (b) you know the nonpivot and pivot variables. Write out the general form of the row reduced echelon matrix and choose the nonzero entries so that the stated vectors form the translation and spanning vectors for the corresponding system.)

10. Redo Exercise 9, using

$$T = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad X_1 = \begin{bmatrix} a \\ b \\ 1 \\ 0 \end{bmatrix}, \quad X_2 = \begin{bmatrix} c \\ d \\ 0 \\ 1 \end{bmatrix}$$

11. Redo Exercise 9 using the variables $x_1, \ldots, x_5$ and

$$T = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad X_1 = \begin{bmatrix} a \\ b \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad X_2 = \begin{bmatrix} c \\ d \\ 0 \\ 1 \\ 0 \end{bmatrix}, \quad X_3 = \begin{bmatrix} e \\ f \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

12. The following vectors T and X_i are, respectively, the spanning and translation vectors for a system of four linear equations. Write the row reduced form of the augmented matrix for the system. (*Hint:* First determine which are the pivot variables. Then write the most general RREF matrix of the appropriate size having these variables as pivots. Compare the spanning and translation vectors for the general system with those given.)

(a) ✓

$$T = \begin{bmatrix} -7 \\ 0 \\ 3 \\ 0 \end{bmatrix}, \quad X_1 = \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad X_2 = \begin{bmatrix} 1 \\ 0 \\ 5 \\ 1 \end{bmatrix}$$

(b)

$$T = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad X_1 = \begin{bmatrix} -1 \\ 0 \\ 4 \\ 1 \\ 0 \end{bmatrix}, \quad X_2 = \begin{bmatrix} -6 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

(c)

$$T = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad X_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad X_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

13. In Example 2, the nonzero entries of the translation vector are the same as the nonzero entries of the last column of the row reduced echelon form. (a) What determines the positions of the 0 entries? Explain how, in general, to determine the translation vector from the last column of the reduced echelon form R of the system together with a knowledge of the positions of the pivot variables. (b) Explain how to determine the entries of the kth spanning vector in terms of the entries of the column of R corresponding to the kth nonpivot variable.

Remark. Exercises 9 – 13 demonstrate specific instances of the result that the translation and spanning vectors uniquely determine the row reduced echelon form of the augmented matrix for the system. This result is the basis of the proof of Theorem 4.

14. One of your engineers wrote a program to solve systems of equations using Gaussian elimination. In a "test" case, the program produced the following answers for the spanning vector T and the translation vectors X_1 and X_2. You immediately knew that the program had an error in it. How? (*Hint:* Write the supposed general solution to his system and attempt to decide which are the nonpivot variables.)

$$T = \begin{bmatrix} 1 \\ 0 \\ -3 \\ 0 \end{bmatrix} \quad X_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix} \quad X_2 = \begin{bmatrix} 2 \\ -1 \\ 1 \\ 0 \end{bmatrix}$$

15. A linear system is homogeneous if each of the constants on the right side of each equation is 0—that is, the numbers b_i in the system (1.9) on page 27 in Section1.2 are all 0. What can you conclude from the more unknowns theorem about a homogeneous system that has fewer equations than unknowns?

16. In each of the folowing systems, find conditions on a, b, and c for which the system has solutions:

(a) ✓

$$\begin{aligned} 3x + 2y - z &= a \\ x + y + 2z &= b \\ 5x + 4y + 3z &= c \end{aligned}$$

(b)

$$\begin{aligned} -3x + 2y + 4z &= a \\ -x - 2y + 3z &= b \\ -x - 6y + 23z &= c \end{aligned}$$

(c) ✓

$$\begin{aligned} 4x - 2y + 3z &= a \\ 2x - 3y - 2z &= b \\ 4x - 2y + 3z &= c \end{aligned}$$

17. Find conditions on a, b, c, and d for which the

$$\begin{aligned} 2x + 4y + z + 3 \\ -3x + y + 2z - 2 \\ 13x + 5y - 4z + 12 \\ 12x + 10y - z + 13 \end{aligned}$$

18. The coefficient matrix (the augmented matrix
system in Exercise 16a is the matrix A belov
where A_i are the rows of A. On the other han
this system is equivalent to $c = a + 2b$, which
Explain this "coincidence." (*Hint:* Compute t
second equations in the system in Exercise 16
relationship between the rows of the coefficie
conditions for the systems in Exercises 16b an

$$A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \\ 5 & 4 \end{bmatrix}$$

19. Show that the consistency conditions found
in terms of linear combinations of the rows c
matrix, just as in Exercise 18.

20. ✓ You are given a vector B and vectors X_i. In
in the span of the X_i by attempting to solve the
x_3X_3.

(a) $B = [3, 2, 1]^t$, $X_1 = [1, 0, -1]^t$, $X_2 = [$

(b) $B = [a, b, c]^t$, $X_1 = [1, 0, -1]^t$, $X_2 = [$

(c) $B = [3, 2, 1]^t$, $X_1 = [1, 0, -1]^t$, $X_2 = [$

21. In a vector space, we say that a particular set of
vector in the space may be written as a linear c
Show that the vectors $X = [1, 2]^t$ and $Y =$
show that the equation $[a, b]^t = xX + yY$ is

22. ✓ Show that the vectors $X_1 = [3, 1, 5]^t$, $X_2 =$
do not span $\mathbb{R}^3$ by finding a vector that canno
nation of them.

23. Prove that a consistent system of five equatio
infinite number of solutions if and only if the
augmented matrix has at least one row of zeros
of nonpivot variables in the echelon form.)

24. Prove that a consistent system of n equatio
infinite number of solutions if and only if the

17. Find conditions on a, b, c, and d for which the following system has solutions:

$$\begin{aligned} 2x + 4y + z + 3w &= a \\ -3x + y + 2z - 2w &= b \\ 13x + 5y - 4z + 12w &= c \\ 12x + 10y - z + 13w &= d \end{aligned}$$

18. The coefficient matrix (the augmented matrix without its last column) for the system in Exercise 16a is the matrix A below. Note that $A_3 = A_1 + 2A_2$, where A_i are the rows of A. On the other hand, the consistency condition for this system is equivalent to $c = a + 2b$, which is essentially the same equation. Explain this "coincidence." (*Hint:* Compute the sum of the first and twice the second equations in the system in Exercise 16a.) Check that there is a similar relationship between the rows of the coefficient matrices and the consistency conditions for the systems in Exercises 16b and 16c.

$$A = \begin{bmatrix} 3 & 2 & -1 \\ 1 & 1 & 2 \\ 5 & 4 & 3 \end{bmatrix}$$

19. Show that the consistency conditions found in Exercise 17 are explainable in terms of linear combinations of the rows of the corresponding coefficient matrix, just as in Exercise 18.

20. ✓You are given a vector B and vectors X_i. In each part, decide whether B is in the span of the X_i by attempting to solve the equation $B = x_1X_1 + x_2X_2 + x_3X_3$.

(a) $B = [3,2,1]^t$, $X_1 = [1,0,-1]^t$, $X_2 = [1,2,1]^t$, $X_3 = [1,1,1]^t$
(b) $B = [a,b,c]^t$, $X_1 = [1,0,-1]^t$, $X_2 = [1,2,1]^t$, $X_3 = [1,1,1]^t$
(c) $B = [3,2,1]^t$, $X_1 = [1,0,-1]^t$, $X_2 = [1,2,1]^t$, $X_3 = [3,4,1]^t$

21. In a vector space, we say that a particular set of vectors "spans the space" if every vector in the space may be written as a linear combination of the given vectors. Show that the vectors $X = [1,2]^t$ and $Y = [1,-2]^t$ span $\mathbb{R}^2$. Specifically, show that the equation $[a,b]^t = xX + yY$ is solvable, regardless of a and b.

22. ✓Show that the vectors $X_1 = [3,1,5]^t$, $X_2 = [2,1,4]^t$, and $X_3 = [-1,2,3]^t$ *do not* span $\mathbb{R}^3$ by finding a vector that cannot be expressed as a linear combination of them.

23. Prove that a consistent system of five equations in five unknowns will have an infinite number of solutions if and only if the row reduced echelon form of the augmented matrix has at least one row of zeros. (*Hint:* Think about the number of nonpivot variables in the echelon form.)

24. Prove that a consistent system of n equations in n unknowns will have an infinite number of solutions if and only if the row reduced echelon form of the

augmented matrix has at least one row of zeros. (*Hint:* Think about the numbe of nonpivot variables in the echelon form.)

25. ✓One possible reduced echelon form of a nonzero 2×2 matrix is shown below What are the other possibilities?

$$\begin{bmatrix} 1 & * \\ 0 & 0 \end{bmatrix}$$

26. One possible reduced echelon form of a nonzero 3×3 matrix is shown below What are the other possibilities?

$$\begin{bmatrix} 1 & * & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

27. Prove that the set of nonzero rows of any matrix having the form of the left matri in formula (1.27) is linearly independent. (*Hint:* See the hint for Exercise 1 on page 17 in Section 1.1.)

28. Repeat Exercise 27 for a matrix of the form shown on the right in formula (1.27 on page 47.

29. What general feature makes it clear that in any matrix in row reduced echelo form, the set of nonzero rows is linearly independent?

30. If you begin with a 5×3 matrix and row reduce until echelon form is obtained is it possible that all the rows of the echelon form are nonzero? If not, wha is the smallest number of zero rows you can get? Explain. Give (if possible examples of 5×3 matrices in echelon form that represent (a) an inconsistent system, (b) a system with exactly one solution, (c) a system with exactly tw solutions, and (d) a system with an infinite number of solutions.

31. ✓In any linear system, the last column of the augmented matrix is called th "vector of constants," while the matrix obtained from the augmented matri by deleting the last column is called the "coefficient" matrix. For example, i Exercise 16a, the vector of constants is $[a, b, c]^t$, which is the vector forme from the constants on the right sides of the equations in this system and th coefficient matrix is

$$\begin{bmatrix} 3 & 2 & -1 \\ 1 & 1 & 2 \\ 5 & 4 & 3 \end{bmatrix}$$

In applications, it often happens that the coefficient matrix remains fixed whil the vector of constants changes periodically. We shall say that the coefficie matrix is "nonsingular" if there is one and only one solution to the system regardless of the value of the vector of constants. [6]

[6] This exercise is intended as a "preview" of the concept of invertibility that is discussed in depth i Chapter 3.

(a) Is the coefficient matrix for the system i plain.

(b) Suppose a given system has three equa nonsingular coefficient matrix. Descri augmented matrix as explicitly as possi

(c) Can a system with two equations and thr coefficient matrix? Explain in terms system.

(d) Can a system with four equations and th coefficient matrix? Explain in terms system.

(e) Can a system with a nonsquare coeffi coefficient matrix? Explain in terms system. (A "square matrix" is one that columns.)

32. Suppose that $ad - bc = 0$. Show that the r dependent.

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

33. ✓In the following system, show that for any c only one) choice of z and w that solves the sy in parametric form using x and y as free vari

$$\begin{aligned} x + z + w \\ y + 2z + w \end{aligned}$$

34. Can x and y be set arbitrarily in this system values.)

$$\begin{aligned} x + 2z + 2w \\ y + z + w \end{aligned}$$

Counting Flops: A rough measure of the take to do a certain task is the total number and /) required. Each such operation is calle The next exercise shows that it requires at m to solve a system of n equations in n unkno unknowns, the solution could require 5,910 681,550 might be required. Such large sys "real-world" applications.

35. Let A be an $n \times (n+1)$ matrix whose row red rows.

(a) Is the coefficient matrix for the system in Exercise 16a nonsingular? Explain.

(b) Suppose a given system has three equations in three unknowns with a nonsingular coefficient matrix. Describe the row reduced form of the augmented matrix as explicitly as possible.

(c) Can a system with two equations and three unknowns have a nonsingular coefficient matrix? Explain in terms of the row reduced form of the system.

(d) Can a system with four equations and three unknowns have a nonsingular coefficient matrix? Explain in terms of the row reduced form of the system.

(e) Can a system with a nonsquare coefficient matrix have a nonsingular coefficient matrix? Explain in terms of the row reduced form of the system. (A "square matrix" is one that has the same number of rows as columns.)

32. Suppose that $ad - bc = 0$. Show that the rows of the matrix A are linearly dependent.

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

33. ✓In the following system, show that for any choice of x and y, there is one (and only one) choice of z and w that solves the system. Write the general solution in parametric form using x and y as free variables.

$$\begin{aligned} x + z + w &= 0 \\ y + 2z + w &= 0 \end{aligned}$$

34. Can x and y be set arbitrarily in this system? (If you are not sure, try a few values.)

$$\begin{aligned} x + 2z + 2w &= 0 \\ y + z + w &= 0 \end{aligned}$$

Counting Flops: A rough measure of the amount of time a computer will take to do a certain task is the total number of algebraic operations ($+$, $-$, $\times$, and $/$) required. Each such operation is called a **flop** (floating point operation.) The next exercise shows that it requires at most $2n^3/3 + 3n^2/2 - 7n/6$ flops to solve a system of n equations in n unknowns. Thus, for example, with 20 unknowns, the solution could require 5,910 flops, while with 100 unknowns, 681,550 might be required. Such large systems are not at all uncommon in "real-world" applications.

35. Let A be an $n \times (n+1)$ matrix whose row reduced echelon form has no nonzero rows.