Math 261
Take Home Exam 1

Please show enough of your work so that I can follow your reasoning process. If you use a calculator tell me what you did. This is an individual exam. You cannot get help from tutors, classmates, friends or anyone other than me. This exam is due Monday October 27th at 6:00pm sharp!

1) Definitions: (2.5 points each)

a. Let $S = A_{1,} A_{2,.} .. A_k$ be a set of elements from some vector space. An element $C = b_1 A_1 + b_2 A_2 + \ldots b_k A_k$ is called a ____________________ of the A_i.

b. The set of all linear combinations of a set of elements is called the __________ of that set.

c. The matrix that we associate with a system of linear equations is called the _________________ matrix.

d. The __________ of a system of equations is the number of rows remaining in the augmented matrix after eliminating linearly independent rows one at a time until a matrix with linearly independent rows is obtained.

e. The span of the columns of a matrix is called the _______________________ of the matrix.

f. The solution space to the homogeneous system $AX = 0$ is called the _______________________ of the matrix A.

g. A ___________ for a vector space is a linearly independent set of elements that spans the vector space

h. The ________________ of a vector space W, is the smallest number of elements it takes to span W.

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2) Gaussian Elimination:

Consider the following system of equations:

$$2x_1 - 7x_2 + 5x_3 - 3x_4 = -4$$
$$x_1 - 2x_2 + x_3 = 1$$
$$x_2 - x_3 + x_4 = 2$$

a) Write the augmented matrix, A (5 points)

b) Here is a partial row reduction of the matrix A

$$\begin{bmatrix} 2 & -7 & 5 & -3 & -4 \\ 0 & 1 & -1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Use elementary row operations to to find the equivalent reduced row echelon form of A. You can not use a calculator on this question. (Show all work.) (10 points)

c) What is the rank of A? Explain. (5 points)

d)Find all solutions to the linear system. Write your answer in parametric form. (5 points)

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3) Column Space and null space

Consider the matrix $A = \begin{pmatrix} 4 & 5 & 3 & 3 \\ 1 & 1 & 1 & 1 \\ 2 & 3 & 1 & 1 \\ 5 & 7 & 3 & 3 \end{pmatrix}$.

a. Show that the columns of A are linearly dependent. (10 points)

b. Find a basis of the column space of A [5 points].

c. Find the null space of A. [10 points]

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Linear Independence:

4) Consider the following set of matrices:

$$S=\left\{\begin{bmatrix}1 & 2\\ 1 & 2\end{bmatrix},\begin{bmatrix}0 & 1\\ 0 & 1\end{bmatrix},\begin{bmatrix}1 & 4\\ 3 & 2\end{bmatrix}\right\}$$

Prove that this set is linearly independent using the the test for linear independence. (15 points)

5) Consider the following set of matrices:

$$S=\{[1,3,-2],[-1,3,3],[-2,0,5]\}$$

a) prove this set is linearly dependent. (15 points)

b) Demonstrate this by explicitly exhibiting one of the elements of the set as a linear combination of the other elements in the set. (10 points)

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Dimension

6) Show that M(2,4) is eight-dimensional (15 points)

Row Space:

7) Let $A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 4 & 4 \\ 3 & 6 & 6 \\ -2 & -4 & -4 \end{pmatrix}$

a) find the rank of A. Explain (10 points)

b) Find a basis for the row space of A (10 points)

c) Find a basis for the column space of A. (5 points)

d) What is the dimension of the null space of A. (5 points)

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8) Use the non-zeros row theorem to find a basis for the space, W, spanned by the matrices: (15 points)

$$[2,3,1,2]^t, [5,2,1,2]^t, [1,-4,-1,-2]^t, [11,0,1,2]^t$$

b) What is the dimension of W? (5 points)

Rank Nullity Theorem

9) Consider the following matrix:

$$A = \begin{bmatrix} 7 & 2 & -5 & 8 \\ -5 & -3 & 4 & -9 \\ 6 & 10 & -2 & 7 \end{bmatrix}$$

a) What is the rank(A). Explain your answer? (10 points)

b) What is the rank of null (A)? Explain your answer (5 points)

d) Is the equation Ax=b solvable for all b in R^3? (5 points)

e) Is the solution to Ax=b unique? (5 points)

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