Solving Linear Systems with Maple:

In this tutorial we will look at a few options in solving the following linear system:

1) x+2y-3z=5
2) y+4z=2
3) 2x-y+z=1

1. Most of the linear algebra functions are included in the linalg or LinearAlgebra package. You must call one of these two packages writing

with(*linalg*)

[*BlockDiagonal, GramSchmidt, JordanBlock, LUdecomp, QRdecomp, Wronskian, addcol, addrow, adj, adjoint, angle, augment, backsub, band, basis, bezout, blockmatrix, charmat, charpoly, cholesky, col, coldim, colspace, colspan, companion, concat, cond, copyinto, crossprod, curl, definite, delcols, delrows, det, diag, diverge, dotprod, eigenvals, eigenvalues, eigenvectors, eigenvects, entermatrix, equal, exponential, extend, ffgausselim, fibonacci, forwardsub, frobenius, gausselim, gaussjord, geneqns, genmatrix, grad, hadamard, hermite, hessian, hilbert, htranspose, ihermite, indexfunc, innerprod, intbasis, inverse, ismith, issimilar, iszero, jacobian, jordan, kernel, laplacian, leastsqrs, linsolve, matadd, matrix, minor, minpoly, mulcol, mulrow, multiply, norm, normalize, nullspace, orthog, permanent, pivot, potential, randmatrix, randvector, rank, ratform, row, rowdim, rowspace, rowspan, rref, scalarmul, singularvals, smith, stackmatrix, submatrix, subvector, sumbasis, swapcol, swaprow, sylvester, toeplitz, trace, transpose, vandermonde, vecpotent, vectdim, vector, wronskian*] **(1)**

or

with(*LinearAlgebra*)

[*&x, Add, Adjoint, BackwardSubstitute, BandMatrix, Basis, BezoutMatrix, BidiagonalForm, BilinearForm, CharacteristicMatrix, CharacteristicPolynomial, Column, ColumnDimension, ColumnOperation, ColumnSpace, CompanionMatrix, ConditionNumber, ConstantMatrix, ConstantVector, Copy, CreatePermutation, CrossProduct, DeleteColumn, DeleteRow, Determinant, Diagonal, DiagonalMatrix, Dimension, Dimensions, DotProduct, EigenConditionNumbers, Eigenvalues, Eigenvectors, Equal, ForwardSubstitute, FrobeniusForm, GaussianElimination, GenerateEquations, GenerateMatrix, Generic, GetResultDataType, GetResultShape, GivensRotationMatrix, GramSchmidt, HankelMatrix, HermiteForm, HermitianTranspose, HessenbergForm, HilbertMatrix, HouseholderMatrix, IdentityMatrix, IntersectionBasis, IsDefinite, IsOrthogonal, IsSimilar, IsUnitary, JordanBlockMatrix, JordanForm, KroneckerProduct, LA_Main, LUDecomposition, LeastSquares, LinearSolve, Map, Map2, MatrixAdd, MatrixExponential, MatrixFunction, MatrixInverse, MatrixMatrixMultiply, MatrixNorm, MatrixPower, MatrixScalarMultiply, MatrixVectorMultiply, MinimalPolynomial, Minor, Modular, Multiply, NoUserValue, Norm,* **(2)**

Normalize, NullSpace, OuterProductMatrix, Permanent, Pivot, PopovForm, QRDecomposition, RandomMatrix, RandomVector, Rank, RationalCanonicalForm, ReducedRowEchelonForm, Row, RowDimension, RowOperation, RowSpace, ScalarMatrix, ScalarMultiply, ScalarVector, SchurForm, SingularValues, SmithForm, StronglyConnectedBlocks, SubMatrix, SubVector, SumBasis, SylvesterMatrix, ToeplitzMatrix, Trace, Transpose, TridiagonalForm, UnitVector, VandermondeMatrix, VectorAdd, VectorAngle, VectorMatrixMultiply, VectorNorm, VectorScalarMultiply, ZeroMatrix, ZeroVector, Zip]

2. Generating the augmented matrix

a.) generating by writing matrix

$A := Matrix([[1, 2, -3, 5], [0, 1, 4, 2], [2, -1, 1, 1]])$

$$\begin{bmatrix} 1 & 2 & -3 & 5 \\ 0 & 1 & 4 & 2 \\ 2 & -1 & 1 & 1 \end{bmatrix} \quad \textbf{(3)}$$

b), Generate using GenerateMatrix

$eqns := \{x + 2y - 3z = 5, y + 4z = 2, 2x - y + z = 1\}$

$$\{y + 4z = 2, x + 2y - 3z = 5, 2x - y + z = 1\} \quad \textbf{(4)}$$

$a, b := GenerateMatrix(eqns, [x, y, z])$

$$\begin{bmatrix} 0 & 1 & 4 \\ 1 & 2 & -3 \\ 2 & -1 & 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix} \quad \textbf{(5)}$$

$augm := \langle a|b \rangle$

$$\begin{bmatrix} 0 & 1 & 4 & 2 \\ 1 & 2 & -3 & 5 \\ 2 & -1 & 1 & 1 \end{bmatrix} \quad \textbf{(6)}$$

3. Solving the linear system using the row reduced echelon form

a) using linalg

$solution := rref(augm)$

$$\begin{bmatrix} 1 & 0 & 0 & \frac{38}{27} \\ 0 & 1 & 0 & \frac{50}{27} \\ 0 & 0 & 1 & \frac{1}{27} \end{bmatrix} \quad \textbf{(7)}$$

b) using LinearAlgebra

$solution2 := ReducedRowEchelonForm(A)$

$$\begin{bmatrix} 1 & 0 & 0 & \frac{38}{27} \\ 0 & 1 & 0 & \frac{50}{27} \\ 0 & 0 & 1 & \frac{1}{27} \end{bmatrix} \quad \textbf{(8)}$$

4. Solve using the equations generated in (4)

$solve(eqns)$

$$\left\{x = \frac{38}{27}, y = \frac{50}{27}, z = \frac{1}{27}\right\} \quad \textbf{(9)}$$

5) Solve using Linear Solve using the matrices generated in (5)

$LinearSolve(a, b)$

$$\begin{bmatrix} \frac{38}{27} \\ \frac{50}{27} \\ \frac{1}{27} \end{bmatrix} \quad \textbf{(10)}$$

6) Performing Row Operations

We will perform row operations to solve A where A=

A

$$\begin{bmatrix} 1 & 2 & -3 & 5 \\ 0 & 1 & 4 & 2 \\ 2 & -1 & 1 & 1 \end{bmatrix} \quad \textbf{(11)}$$

Notation:

a) Swap Ri with Rj= A:=RowOperation(A,[i,j])

example: swap row 2 and row 3

$A := RowOperation(A, [2, 3])$

$$\begin{bmatrix} 1 & 2 & -3 & 5 \\ 2 & -1 & 1 & 1 \\ 0 & 1 & 4 & 2 \end{bmatrix} \quad \textbf{(12)}$$

b) Add a multiple of one row to another: A:=RowOperation(A,[j,i],c)

example: add -2*r1 to r2

$A := RowOperation(A, [2, 1], -2)$

$$\begin{bmatrix} 1 & 2 & -3 & 5 \\ 0 & -5 & 7 & -9 \\ 0 & 1 & 4 & 2 \end{bmatrix} \quad \textbf{(13)}$$

c) Multiply a row by constant: A:=RowOperation(A,i,c)

example: multiply row 2 by (-1/5)

$A := RowOperation\left(A, 2, -\frac{1}{5}\right)$

$$\begin{bmatrix} 1 & 2 & -3 & 5 \\ 0 & 1 & -\frac{7}{5} & \frac{9}{5} \\ 0 & 1 & 4 & 2 \end{bmatrix} \quad \textbf{(14)}$$