Spotted owl Problem: Project 2:

Background: 1990 Northern spotted owl
become center of Nationwide controversy
over the use of the Pacific Northwest Forest.

→ Environmentalist argued that the spotted owl was threatened with extinction.

→ Timber industry anticipating the loss of 30,000 → 100,000 jobs argued that the spotted owl should not be on the endangered species list.

Mathematical ecologist - Rollie Lamberson (Humboldt State)
figured out a mathematical model to describe the dynamics of the spotted owl.

Formulation

★ In population models the # male, # female are assumed even, and the female population is only considered.

★ The spotted owl can naturally broken down into three age cohorts:

(J)	Juvenile	(0 - 1 years)	do not mate
(S)	Sub-adult	(1 - 2 years)	breeding rare
(A)	Adults	(2 +)	breed.

A common way to model population is with a Leslie - Matrix

Dynamics

$$J_{t+1} = F_S S_t + F_a S_A$$
$$S_{t+1} = P_j J_t$$
$$A_{t+1} = P_S S_{t+1} + P_A A_{t+1}$$

$$\Rightarrow \begin{bmatrix} J_{t+1} \\ S_{t+1} \\ A_{t+1} \end{bmatrix} = \begin{bmatrix} 0 & F_s & F_a \\ P_j & 0 & 0 \\ 0 & P_S & P_A \end{bmatrix} \begin{bmatrix} J_t \\ S_t \\ A_t \end{bmatrix}$$

where

F_S = Fecundity rate of sub adults

F_a = Fecundity rate of adults

P_j = Probability of Juvenile survival

P_S = probability of subadult survival

P_j = probability of Adult survival

Using a large amount of Data the following Statistics were found:

$F_S \approx 0$

$F_a = .33$

$P_j = .18$

$P_S = .71$

$P_A = .94$

Therefore the dynamics become:

$$\begin{bmatrix} J_{t+1} \\ S_{t+1} \\ A_{t+1} \end{bmatrix} = \begin{bmatrix} 0 & 0 & .33 \\ .18 & 0 & 0 \\ 0 & .71 & .94 \end{bmatrix} \begin{bmatrix} J_t \\ J_t \\ A_t \end{bmatrix}$$

We can compute this dynamic system for several time steps to see the population is decreasing toward zero.

Graphing in Excel

First construct Leslie Matrix:

we will Assume initial population

$$[J_0, S_0, A_0] = [10{,}000,\ 4000, 25000]$$

write out the matrix vector calculation to determine $[J_1, S_1, A_1]$

where =

$$J_1 = (J_0 \cdot 0) + (S_0 \cdot F_S) + (A_0 \cdot F_A)$$
$$S_1 = J_0 \cdot P_j + (S_0 \cdot 0) + (A_0 \cdot 0)$$
$$A_1 = J_0 \cdot 0 + S_0 \cdot P_S + A_0 \cdot P_A$$

put \$ infront of the numbers and letters of the indices from the Leslie Matrix.

Fill down to get the results for more time steps.

* Graph the dynamics

Again we can see the population is declining:

We can find the closed form for these dynamics by finding the eigen values, eigen vectors, for the Leslie Matrix.

Find the eigenvectors, eigen values of:

$$A = \begin{bmatrix} 0 & 0 & .33 \\ .18 & 0 & 0 \\ 0 & .71 & .94 \end{bmatrix}$$

How do we do this:

$$Ax = \lambda x \Rightarrow Ax - \lambda x = 0$$
$$\Rightarrow (A - \lambda I)x = 0$$

TO FIND NON-TRIVIAL SOLUTION we find

$$\det(A - \lambda I) = 0$$

FIND $\text{Det} \begin{bmatrix} -\lambda & 0 & .33 \\ .18 & -\lambda & 0 \\ 0 & .71 & .94-\lambda \end{bmatrix} = 0$

$$= -\lambda \begin{vmatrix} -\lambda & 0 \\ .71 & .94-\lambda \end{vmatrix} + .33 \begin{vmatrix} .18 & -\lambda \\ 0 & .71 \end{vmatrix} = 0$$

$$= -\lambda(-\lambda(.94-\lambda)) + .33(.18 \cdot .71) = 0$$

$$= .94\lambda^2 - \lambda^3 + .0422 = 0$$

$$= \lambda^3 + .94\lambda^2 + .0422 = 0$$

Tough to factor
USE calculator or Maple to find eigenvalues, eigenvectors

From Maple we determine:

$\lambda_1 = .9836$ .9836 $\quad \bar{V}_1 = [0.3175, 0.0581, 0.9465]$

$\lambda_2 = -0.0218 - 0.2059i \quad \bar{V}_2 = [0.6821, -0.0624 + .5896i, -0.0451 - .4256i]$

$\lambda_3 = -0.0218 + 0.2059i \quad \bar{V}_3 = [0.6821, -0.0624 - .5896i, -0.0451 + .4256i]$

Finding closed form solution:

we know

$$A\bar{V}_1 = \lambda_1 V_1 \;,\; A\bar{V}_2 = \lambda_2 \bar{V}_2 \;,\; A\bar{V}_3 = \lambda_3 \bar{V}_3$$

we want to write x_0 in terms of span of eigenspace

$$x_0 = \begin{bmatrix} 10000 \\ 4000 \\ 25000 \end{bmatrix} = a \begin{vmatrix} v_1 \end{vmatrix} + b \begin{vmatrix} v_2 \end{vmatrix} + c \begin{vmatrix} v_3 \end{vmatrix}$$

Augmented matrix

$$\begin{vmatrix} v_1 & v_2 & v_3 & x_0 \end{vmatrix} = \begin{bmatrix} 0.3175 & 0.6821 & 0.6821 & 10000 \\ 0.0581 & -0.0624 + .5896i & -0.0624 - .5896i & 4000 \\ 0.9465 & -0.0451 - .4256i & -0.0451 + .4256i & 25000 \end{bmatrix}$$

rref $\Rightarrow$

$$\begin{bmatrix} 1 & 0 & 0 & 28347 \\ 0 & 1 & 0 & 733 - 2073i \\ 0 & 0 & 1 & 733 + 2073i \end{bmatrix}$$

check

$$\begin{bmatrix} 10001 \\ 4000 \\ 24999 \end{bmatrix} = 28347 \begin{bmatrix} V_1 \end{bmatrix} + 733 - 2073i \begin{bmatrix} V_2 \end{bmatrix} + 733 + 2073i \begin{bmatrix} V_3 \end{bmatrix}$$

To Find closed form Then:

Let $X_t = \begin{bmatrix} S_t \\ S_t \\ A_t \end{bmatrix}$ and $X_{t+1} = \begin{bmatrix} S_t \\ S_t \\ A_t \end{bmatrix}$ $X_0 = \begin{bmatrix} 10000 \\ 4000 \\ 25000 \end{bmatrix}$

$X_1 = A X_0$

$X_1 = A\left[28347\bar{v}_1 + (733-2073i)\bar{v}_2 + 733+2073i\ \bar{v}_3\right]$

$= 28347 A\bar{v}_1 + (733-2073i)A\bar{v}_2 + 733+2073i\ A\bar{v}_3$

$= 28347\lambda_1 v_1 + (733-2073i)\lambda_2 v_2 + 733+2073i\ \lambda_3\bar{v}_3$

Then

$X_2 = A[X_1]$

$= A\left[28347\lambda_1\bar{v}_1 + (733-2073i)\lambda_2\bar{v}_2 + 733+2073i\ \lambda_3\bar{v}_3\right]$

$= 28347\lambda_1 A\bar{v}_1 + (733-2073i)\lambda_2 A\bar{v}_2 + 733+2073i\ \lambda A\bar{v}_3$

$= 28347\lambda_1^2 v_1 + (733-2073i)\lambda_2^2\bar{v}_2 + (733+2073i\ \lambda_3^2\bar{v}_3)$

In General

$X_n = \left[28347\lambda_1^n v_1 + (733-2073i)\lambda_2^n\bar{v}_2 + (733+2073i)\lambda_3^n\bar{v}_3\right)$

recall

$\lambda_1 = 0.9836 \quad \lim_{n\to\infty}\lambda_1^n \to 0$

For complex numbers we consider $|\lambda_2|$

recall

$z = x+yi$ Then $|z| = \sqrt{x^2+y^2}$

If the magnitude of $|\lambda_2|<1$ Then $\lim_{n\to\infty}\lambda^n \to 0$

$|\lambda_2| = |\lambda_3| = \sqrt{(-0.218)^2 + (-0.2059)^2} = .2071$

Therefore

$$\lim_{n\to\infty} x_n = \lim_{n\to\infty} \left[28347\lambda_1^n v_1 + (733 - 2073i)\lambda_2^n v_2^n + \right]$$

$$= 0v_1 + 0v_2 + 0v_3$$

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

So how do we fix this problem?

Well it turns out that

$$A_1 = \begin{bmatrix} 0 & F_S & F_a \\ P_j & 0 & 0 \\ 0 & P_S & P_A \end{bmatrix}$$

That P_j was the most significant

what factors into $P_j \Rightarrow$

1. Each pair of owls needs $\approx$ 1000 hectares to themselves. (therefore cutting down forest realizes fewer possible habitats) Juveniles die of overcrowding

2. Patch cutting, Clearings make the Juveniles vulnerable to attacks from predators

The logging industry came up with a newmodel

Leslie Matrix

$$\begin{bmatrix} 0 & 0 & .33 \\ .4 & 0 & 0 \\ 0 & .71 & .94 \end{bmatrix}$$

eigenvalues of $A =$

$\lambda_1 = 1.0286$

$\lambda_2 = -0.0443 + 0.2986i$

$\lambda_3 = -0.0443 - 0.2986i$