

Linear equations:

Linear equation in one variable

A linear equation in one variable is an equation that can be written in the form

$$ax + b = 0$$

where a and b are real numbers with $a \neq 0$.

Symbolic solutions

Addition property,

$$a = b \Leftrightarrow a + c = b + c$$

Multiplication property:

$$a = b \Leftrightarrow ac = bc$$

Example:

$$-5(3 - 2x) - (1 - x) = 4(x - 3)$$

$$-15 + 10x - 1 + x = 4x - 12$$

$$11x - 16 = 4x - 12$$

$$7x = 4$$

$$x = \frac{4}{7}$$

check

$$-5\left(3 - 2\left(\frac{4}{7}\right)\right) - \left(1 - \frac{4}{7}\right) = 4\left(\frac{4}{7} - 3\right)$$

$$-15 + \frac{40}{7} - 1 + \frac{4}{7} = \frac{16}{7} - 12$$

$$-14 + \frac{44}{7} = \frac{16}{7} - 12$$

~~$$-14 + \frac{44}{7} = \frac{16}{7} - 12$$~~

$$\frac{28}{7} = 4$$

$$4 = 4$$

Three Types of Equations:

① Contradiction

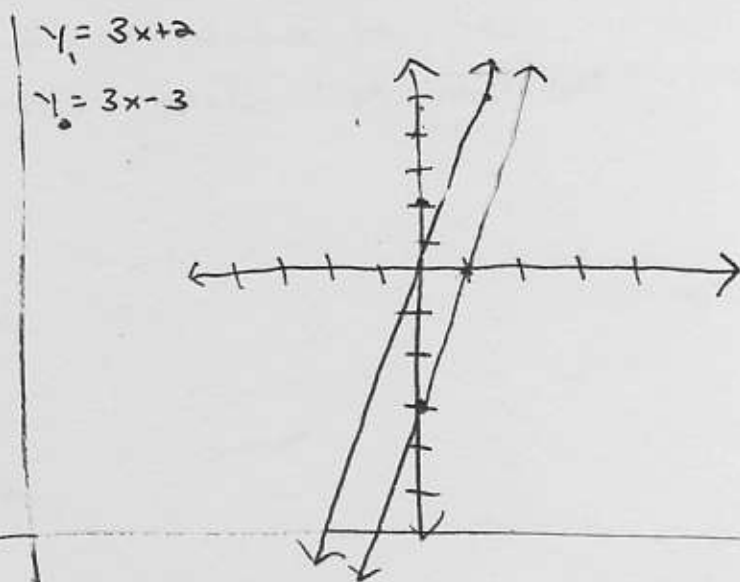
$$3x + 2 = 3x - 3$$

$$0 = -5$$

Contradiction

So no solutions

Parallel - lines no intersection.



② identity

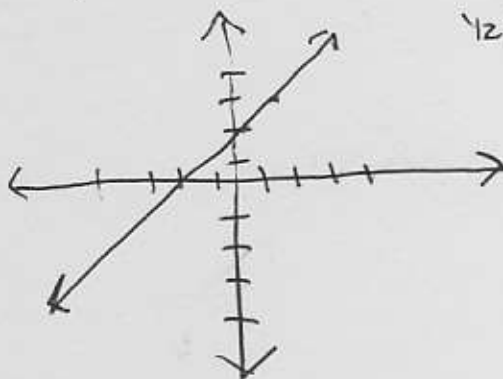


$$x + 2 = \frac{1}{3}(3x + 6)$$

$$3x + 6 = 3x + 6$$

$$0 = 0$$

$$y_1 = x + 2$$
$$y_2 = \frac{1}{3}(3x + 6)$$



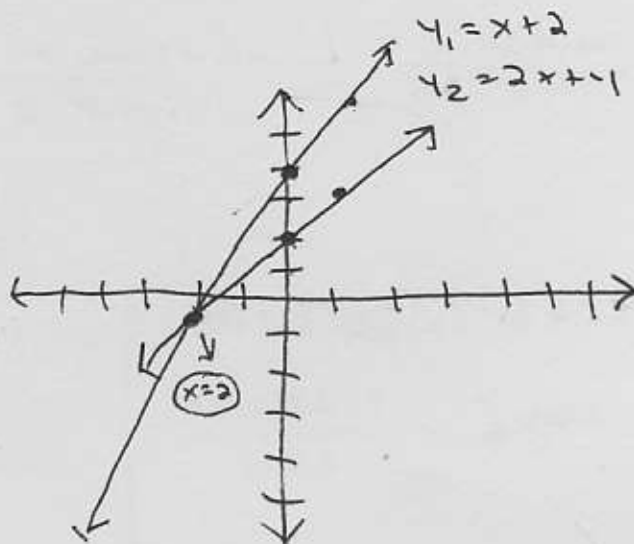
③ Conditional equation

$$x + 2 = 2x + 4$$

$$x - 2x = 4 - 2$$

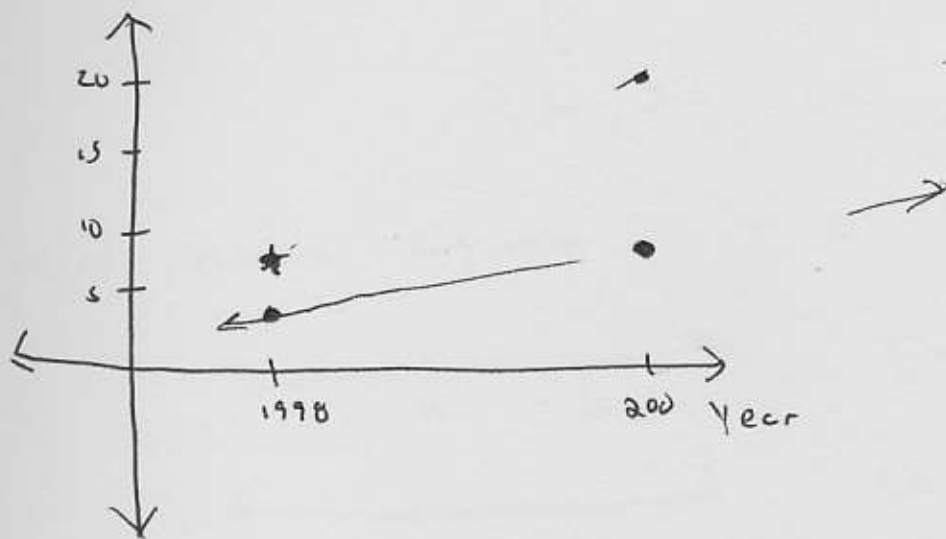
$$-x = 2$$

$$x = -2$$



Application:

In 1998 worldwide sales of PCs were 4 million and in 2000 they were 7.7 million. Use a linear function f to estimate the year when sales could reach 19 million.



point-slope equation:

$$y = y_1 + m(x - x_1)$$

$$m = \frac{7.7 - 4 \text{ million}}{2000 - 1998 \text{ years}} = \frac{7.7 - 4 \text{ years million}}{2 \text{ years}} = \frac{3.7}{2} \frac{\text{million}}{\text{year}} = \frac{1.85 \text{ mill}}{\text{year}}$$

$$y = 4 + 1.85(x - 1998)$$

$$y = 4 + 1.85x - 3692.3$$

$$y = 1.85x - 3692.3$$

$$y = 19$$

$$19 = (1.85)(x) - 3692.3$$

$$\frac{3692.3 + 19}{1.85} = 2006.10$$

Linear Inequalities:

Linear inequality in one variable:

A linear inequality in one-variable is an inequality that can be written in the form

$$ax + b \text{ (} \text{>, <, \geq, \leq \text{)}$$

where $a \neq 0$

$$\text{>, <, \geq, \leq}$$

recall Interval notation



Properties of inequalities:

addition

$$a < b \quad \text{then} \quad a + c < b + c$$

multiplication

① if $c > 0$

$$a < b \quad \text{then} \quad ac < bc$$

② if $c < 0$

$$a < b \quad \text{then} \quad ac > bc$$

↓
flip
inequality

Examples:

15

$$-2(x-10) + 1 > 0$$

$$-2x + 20 + 1 > 0$$

$$-2x + 21 > 0$$

$$-2x > -21$$

$$x < \frac{-21}{-2}$$

$$x < \frac{21}{2}$$

INTERVAL NOTATION

$$\left[-\infty, \frac{21}{2}\right)$$

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$$\frac{3x}{4} < x - \frac{x+2}{4}$$

multiply both sides by LCD

$$\cancel{4} \left(\frac{3x}{\cancel{4}}\right) < \cancel{4} \left(x - \frac{x+2}{\cancel{4}}\right)$$

$$3x < 4x - (x+2)$$

$$3x < 4x - x + 2$$

$$3x < 3x + 2$$

$$0 < 2$$

Always true

So infinite # of solutions

Solving linear equations graphically:

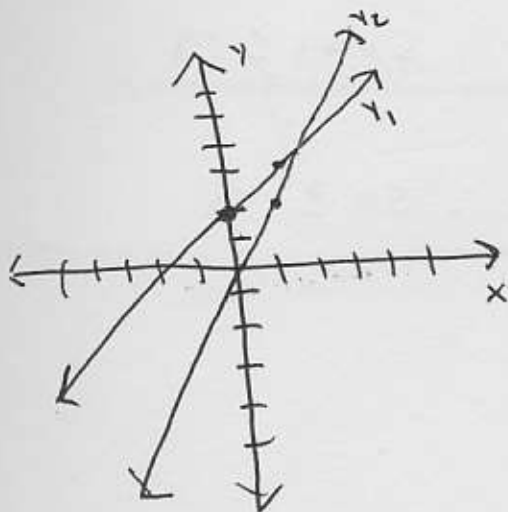
(41)

$$x+2 \geq 2x$$

let

$$y_1 = x+2 > y_1 > y_2$$

$$y_2 = 2x$$



From my graph

$$(-\infty, 1.5)$$

$$(-\infty, 2)$$

To find intersection

$$x+2 = 2x$$

$$2 = x$$

(60) Hw

$$\sqrt{2}x > 10.5 - 13.7x$$

Solve graphically

$$y_1 = \sqrt{2}x$$

$$y_2 = 10.5 - 13.7x$$

Find solution using trace function on calculator.

Compound inequalities:

Sometimes a variable must satisfy
two inequalities

$$-7 \leq 5x+1 < 21$$

Solve each inequality separately

$$-7 \leq 5x+1 < 21$$

$$\Rightarrow 5x+1 < 21 \quad \text{and} \quad 5x+1 \geq -7$$

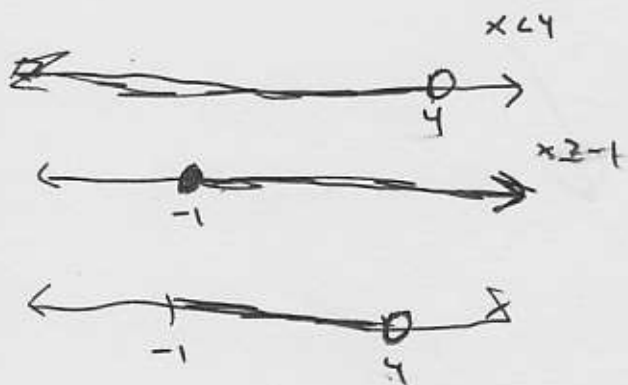
$$5x < 20$$

$$x < 4$$

and

$$5x \geq -8$$

$$x \geq -1$$



$$[-1, 4)$$

(68)

$$-3 \leq 1-x \leq 2x$$

↓

$$1-x \leq 2x \Rightarrow \text{and}$$

$$1-x \geq -3$$

$$1-x \geq -3$$

$$-x \geq -4$$

$$x \leq 4$$

⇒

$$-3x \leq -1$$

$$x \geq 1/3$$

