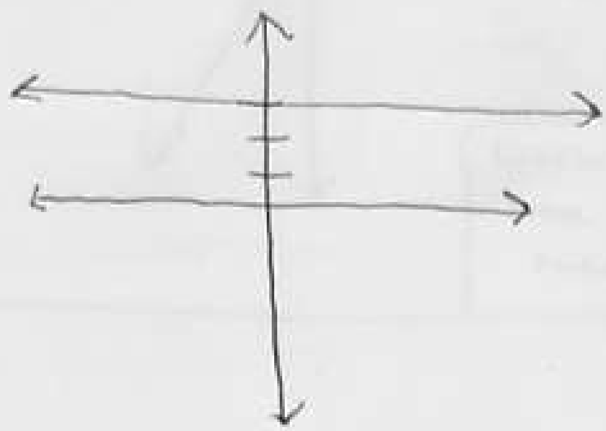
1.4 Types of functions and their rates of change:

Constant functions

A function f represented by $f(x) = b$ where b is a fixed number is a constant function.

ex.

$f(x) = 3$



Domain = $\mathbb{R}$

RANGE = $\{3\}$

No matter what x value I choose my y-value is always 3.

Applications:

Age during a given year $A(t) = 30$ years

Speed of car using cruise control $S(t) = 30$ mph

Temperature of your house $T(t) = 68^{0}$

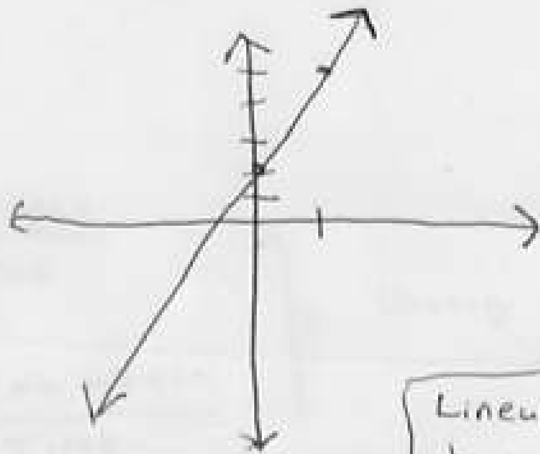
Linear Functions:

A function f represented by $f(x) = ax+b$ where a and b are constants, is a linear function

ex.

$f(x) = 3x + 2$

x	$f(x)$	
0	2	$3(0)+2$
1	5	$3(1)+2$



$\boxed{\text{Linear functions have constant rates of change}}$!

Applications:

constant function: Distance between you and your house during class:

$$D(t) = 15 \text{ miles}$$

Linear function: Distance between you and your house after class driving at 30 mph.

$$D(t) = 15 - t(30) = -30t + 15$$

$$0 \le t \le 1/2$$

-30 is my constant rate of change.

Slope as a rate of change:

rate of change - describes a relationship between one quantity with respect to another

examples:

$$\text{Speed} = \frac{\text{Distance}}{\text{TIME}}$$

$$\text{FLOW} = \frac{\text{Volume of water}}{\text{TIME}}$$

→ Usually with time

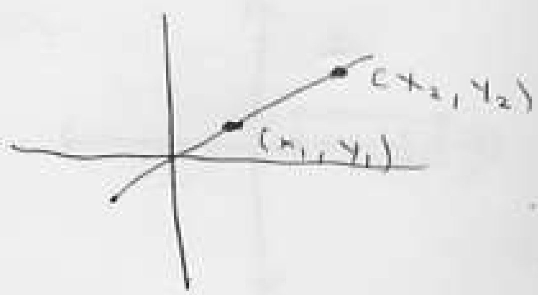
When you are climbing a mountain you may notice

$$\frac{\text{temperature}}{\text{elevation}} \qquad \frac{\text{Amount of oxygen}}{\cancel{\text{distance}}\ \text{elevation}}$$

Now recall the slope of a line is constant:

Slope of a line passing through two points $(x_1, y_1), (x_2, y_2)$ is

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$$

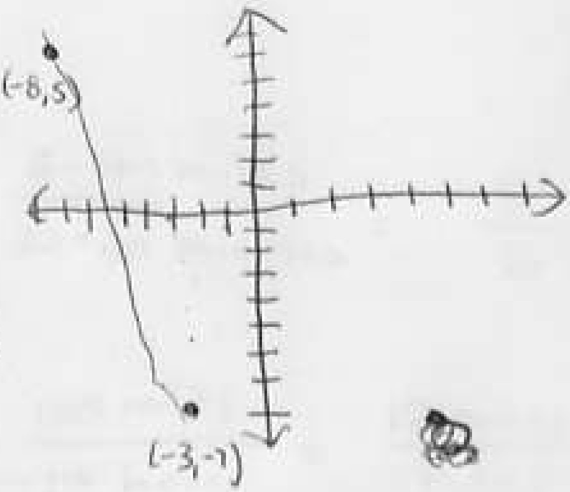


Calculating the slope of a line

passing through the points $\overset{x_1\ \ y_1}{(-8, 5)}$ and $\overset{x_2\ \ y_2}{(-3, -7)}$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-7 - 5}{-3 - (-8)} = \frac{-12}{5} = \frac{-12}{5}$$

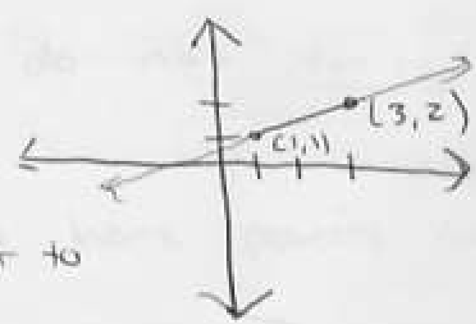
$$= \frac{y_1 - y_2}{x_1 - x_2} = \frac{5 - (-7)}{-8 - (-3)} = \frac{12}{-5} = \frac{-12}{5}$$



negative slope - decreases from left to right

passing through the points (1,1) and (3,2)

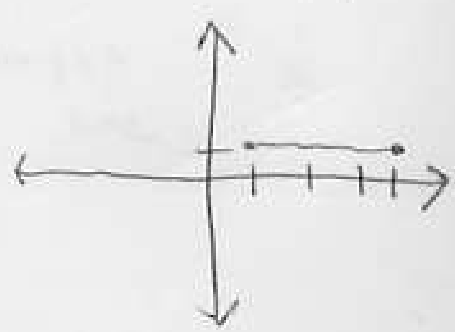
$$m = \frac{3-1}{2-1} = \frac{2}{1} = 2$$



positive slope = increases from left to right

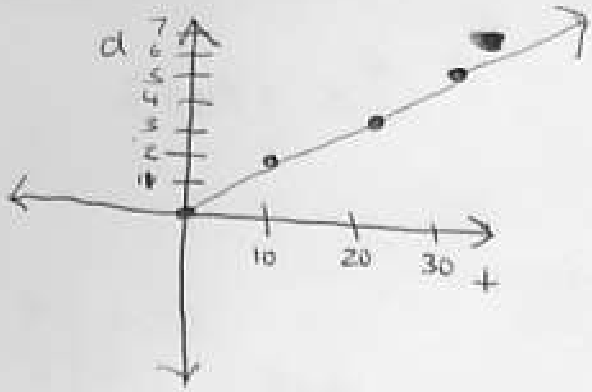
Passing through the points (1,1) and (4,1)

$$M = \frac{1-1}{4-1} = \frac{0}{3} = 0$$



0 slopes = stay the same from left to right

Quick Application: Find the speed of the bicyclist



points (10, 2) = 10 minutes, 2 mil

(20, 4) =

(30, 6) =

slope of line = $m =$ ~~30 − 20~~ $\dfrac{6-4 \text{ miles}}{30-20 \text{ minutes}} = \dfrac{2}{10}\dfrac{\text{miles}}{\text{minutes}}$

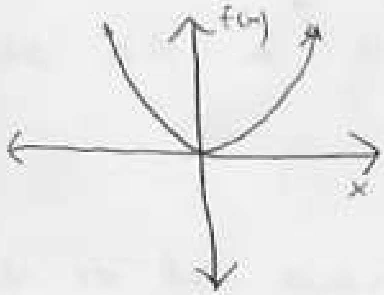
$$\frac{2 \text{ miles}}{10 \cancel{\text{minutes}}}\left(\frac{60 \cancel{\text{minutes}}}{1 \text{ hour}}\right) = \frac{120 \text{ miles}}{12 \text{ hours}} = \frac{12 \text{ miles}}{\text{hour}}$$

Nonlinear functions: A function that is not linear (from our book)

① functions whose graphs do not form straight lines.

② functions whose variables have powers not equal to 1.

ex. $f(x) = x^{2}$ — greater then 1.

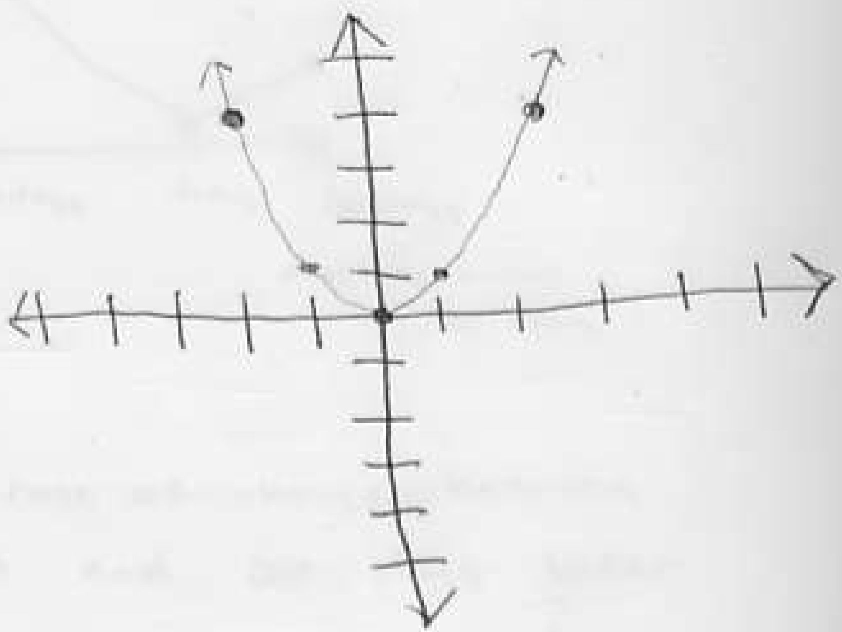


← NOT A straight line

Consider

$$f(x) = x^2$$

x	$f(x)$	
0	0	(0,0)
1	1	(1,1)
-1	1	(-1,1)
2	4	(2,4)
-2	4	(-2,4)



The slope of this curve is always changing. Notice that from negative to zero the slope is negative and from 0 to infinity the slope is positive. So we can't calculate a "single rate of change" but we can calculate an average rate of change.

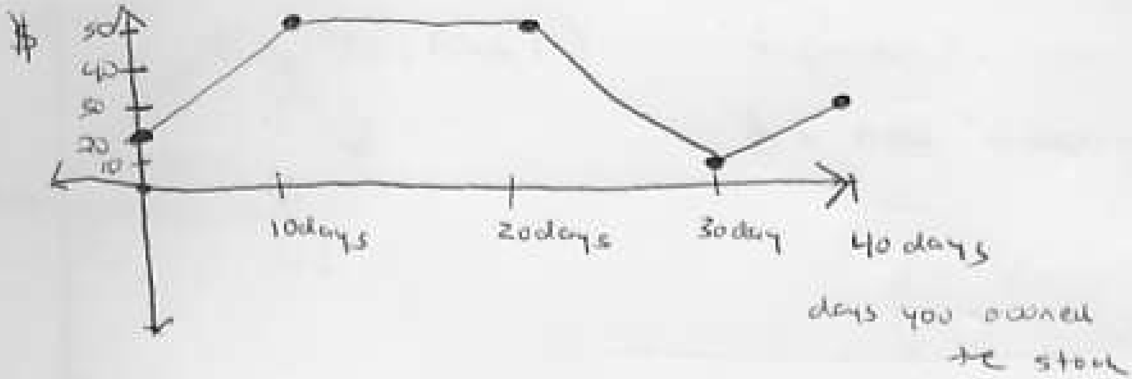
Average rate of change:

Let (x_1, y_1) and (x_2, y_2) be distinct points on the graph of a function. The average rate of f from x_1 to x_2 is

$$ARC = \frac{y_2 - y_1}{x_2 - x_1}$$

That is the average rate of change from x_1 to x_2 equals the slope of the line passing through (x_1, y_1), (x_2, y_2)

Applications: Stock Quote:



What is te average rate of change between when you bought it and 20 days later

$P_1 = (0,20)$, $P_2 = (20,50)$

$$Arc = \frac{(50-20)}{(20-0)} \frac{\text{dollars}}{\text{day}} = \frac{30}{20} = \frac{3}{2} \frac{\text{dollars}}{\text{day}}$$

$+ \boxed{\$1.50 / \text{day}}$

What is te average rate of chage between when you bought it an 30 days later

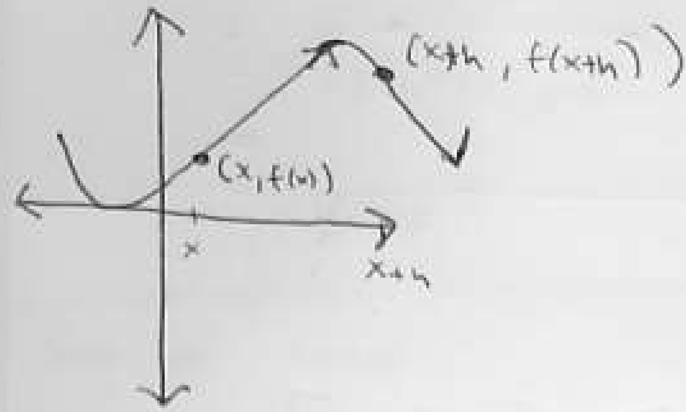
$P_1 = (0,20)$ $P_2 = (30,10)$

$$Arc = \frac{10-20}{30-0} \frac{\text{dollars}}{\text{day}} = \frac{-10}{30} \frac{\text{dollars}}{\text{day}} = \frac{-1}{3} \frac{\text{dollar}}{\text{day}} \approx$$ lost 33¢ /day

What is te average rate of chang between when you bought it an 40 days later

$P_1 = (0,20)$ $P_2 = (40,30)$

A more generalized approach



h is now a variable

<u>Difference Quotient!</u> The difference quotient of a function f is an expression of the form

$$\frac{f(x+h)-f(x)}{h}, \quad h \neq 0$$

Consider

$f(x) = x^2$

Determine

$f(x) = x^2$

$f(x+h) = (x+h)^2 =$

Then

$$\frac{f(x+h) - f(x)}{h}$$

$$= \frac{(x+h)^2 - x^2}{h}$$

$$= \frac{\cancel{x^2} + 2hx + h^2 - \cancel{x^2}}{h}$$

$$= \frac{\cancel{h}(2x+h)}{\cancel{h}} = 2x+h$$

$f(x) = 3x^2 + 1$

FIND THE Difference Quotient:

$f(x) = 3x^2 + 1$

$f(x+h) = 3(x+h)^2 + 1$

So

$$\frac{f(x+h) - f(x)}{h} = \frac{\left[3(x+h)^2 + 1\right] - \left[3x^2 + 1\right]}{h}$$

$$= \frac{3(x^2 + 2hx + h^2) + 1 - 3x^2 - 1}{h}$$

$$= \cancel{3x^2} + 6hx + 3h^2 \cancel{+1} - \cancel{3x^2} \cancel{-1}$$

$$= \frac{6hx + 3h^2}{h} = \frac{h(6x + 3h)}{h}$$

$$\boxed{= 6x + 3h}$$