3.1 Quadratic functions and Models:

Defn.

Let a, b, and c be real numbers with $a \neq 0$. A function represented by

$$f(x) = ax^2 + bx + c$$ is called

a quadratic function.

Defn: The coefficient of the variable with the highest degree is called the leading coefficient

Identifying Quadratic functions:

Homework problems: identify the function as linear, quadratic or Neither:

$f(x) = x + 1$ linear function

leading coefficient = 1

$f(x) = 2x^2 + 3x + 1$

quadratic function leading coefficient = 2

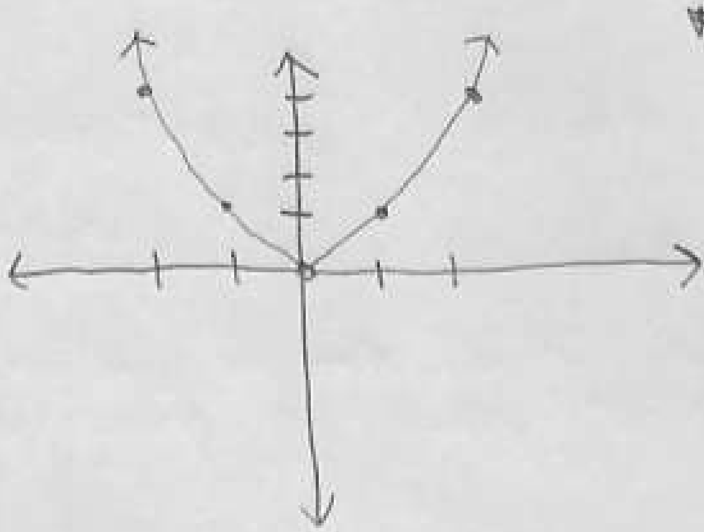
$f(x) = \dfrac{3}{x^2+1}$ = Not linear or quadratic

Graphs of Quadratic functions:

Consider:

$f(x) = x^2$ = Forms a parabola (quadratic function)

x	f(x)
0	0
1	1
-1	1
2	4
-2	4



Vertex = Maximum or Minimum value of the parabola (cartesian coordinates for the)

Vertex = (0, 0)

axis of symmetry = The line where the two sides of the graph on either side look like mirror images of each other

$x = 0$

Vertex form:

The parabolic graph of $f(x) = a(x-h)^2 + k$ with $a \neq 0$ has:

① vertex $= (h, k)$

② axis of symmetry : $x = h$

③ if $a > 0$

  ⓐ opens up

  ⓑ has a minimum value of $y = k$ when $x = k$

④ if $a < 0$

  ⓐ opens downward

  ⓑ has a maximum value of $y = k$ when $x = k$

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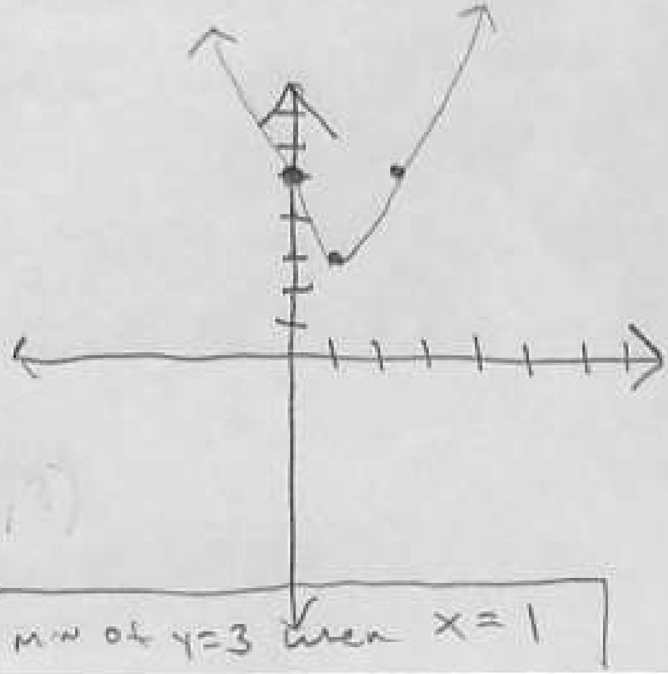
example:

$f(x) = 2(x-1)^2 + 3$



x	f(x)
0	5
1	3
-1	11
2	5
-2	

$2(0-1)^2 + 3$

$a = 2$
$h = 1,\ k = 3$

vertex $= (1, 3)$

opens up ; has a min of $y = 3$ when $x = 1$

examples

(18) Identify the vertex, leading coefficient, axis of symmetry, minimum or maximum value and state if graph opens up or down

(19) $f(x) = 5(x+2)^2 - 5$

$h = -2$ $\Rightarrow$ vertex $= (h,k) = (-2,-5)$

$k = -5$ $\Rightarrow$ l.c $= 5$

$a = 5$ $\Rightarrow$ graph opens up

$\Rightarrow$ minimum value of $y = -5$ when $x = -2$

$\Rightarrow$ axis of symmetry $x = -2$

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(22) $f(x) = -5(x-4)^2$

$h = 4$ $\Rightarrow$ vertex $= (h,k) = (4,0)$

$k = 0$ $\Rightarrow$ l.c $= -5$

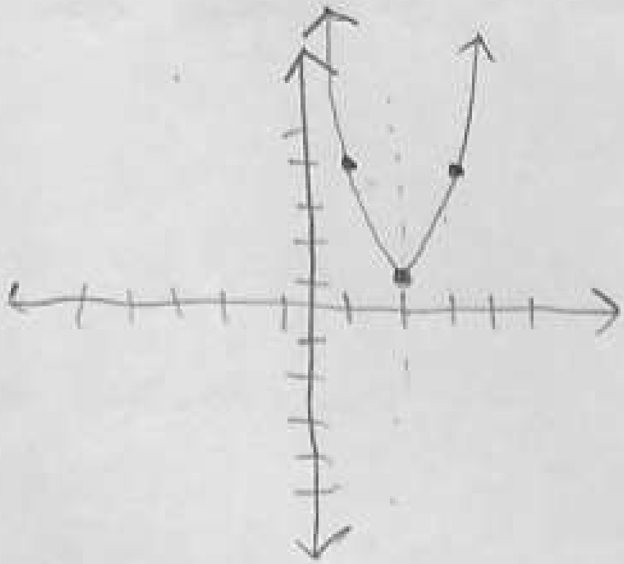
$a = -5$ $\Rightarrow$ graph opens down

$\Rightarrow$ maximum value at $y = 0$ when $x = 0$

$\Rightarrow$ axis of symmetry $x = 4$

Sketching graphs of parabolas!

$f(x) = 3(x-2)+1$



① plot vertex : $(2,1)$

② evaluate one point to right of vertex

x	$f(x)$
2	1
3	4

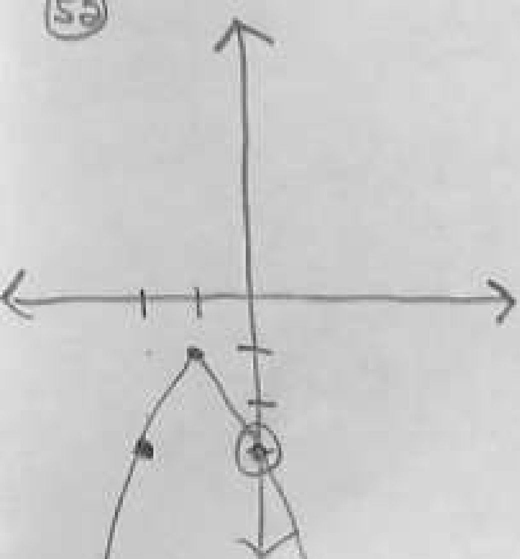
→ vertex

$3(3-2)+1$

③ Use axis of symmetry to evaluate left side of graph

Determining the function $f(x) = a(x-h)^2+k$ using the graph of a parabola:

5a



① Find vertex:

Vertex $= (h,k) = (-1,-1)$

② plug into equation

$f(x) = a(x-(-1))^2 + (-1)$

$f(x) = a(x+1)^2 - 1$

③ use a point to determine a

$(0,-3) = (x, f(x))$

$-3 = a(0+1)^2 - 1$

$-3 = a - 1 \Rightarrow \boxed{a = -2}$

So $\boxed{f(x) = -2(x+1)^2 - 1}$

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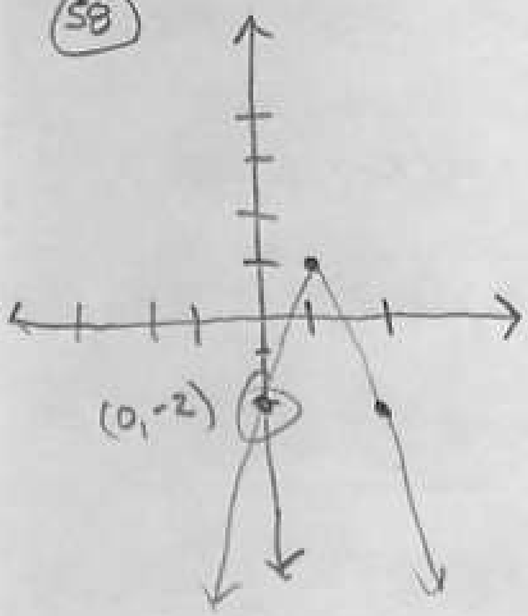


① vertex = $(h,k) = (1,1)$

② $f(x) = a(x-1)^2 + 1$

③ choose point $(0,-2)$

④ Then

$$-2 = a(0-1)^2 + 1$$

$$-2 = a + 1$$

$$-3 = a$$

⑤ $\boxed{f(x) = -3(x-1)^2 + 1}$

So what if quadratic equations are not written in vertex form:

<u>method of completing the square</u>

goal is to write

$f(x) = ax^2 + bx + c$ into

$f(x) = a(x-h)^2 + k$

completing the square: (when leading coefficient is 1)

Consider:

x^2+2x+6 want to write it as

$\Rightarrow ax^2+bx+c$ $(x-h)^2+k$

$b=2$ $f(x) = (x^2+2x)+6$

$\frac{b}{2}=1$ $= (x^2+2x+1)+6-1$

$\left(\frac{b}{2}\right)^2=1$

perfect square

add 1

minus 1

don't change equation

$$\boxed{f(x) = (x+1)^2+5}$$

$$\boxed{f(x) = x^2-7x+5}$$

① isolate x's

$(x^2-7x)+5$

$b=7$

$\frac{b}{2}=\frac{7}{2}$

$\left(\frac{b}{2}\right)^2=\left(\frac{7}{2}\right)^2=\frac{49}{4}$

Add / Subtract $(b/2)^2$

$\left(x^2-7x+\frac{49}{4}\right)+5-\frac{49}{4}$

$\left(x-\frac{7}{2}\right)^2+\frac{20}{4}-\frac{49}{4}$

$$\boxed{f(x) = \left(x-\frac{7}{2}\right)^2-\frac{29}{4}}$$

When te leading coefficient is not 1

$f(x) = 3x^2 + 6x + 2 \Rightarrow f(x) = (3x^2 + 6x) + 2$

pull out leading coefficient

$f(x) = 3(x^2 + 2x) + 2$

$b = 2$

$\frac{b}{2} = 1$

$\left(\frac{b}{2}\right)^2 = 1$

$\Rightarrow f(x) = 3(x^2 + 2x + 1) + 2 - 3$

have actually added 3

need to subtract 3

$\boxed{f(x) = 3(x+1)^2 - 1}$

$f(x) = \left(-\frac{1}{2}x^2 - x\right)$

$f(x) = \left(-\frac{1}{2}x^2 - x\right)$

$= -\frac{1}{2}\left(x^2 + 2x\right)$

$= -\frac{1}{2}\underbrace{(x^2 + 2x + 1)}_{\text{actually subtracted } 1/2} + 1/2$

$b = 2$

$\frac{b}{2} = 1$

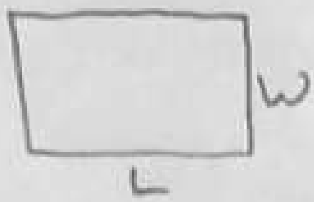
$\left(\frac{b}{2}\right)^2 = 1$

$\boxed{f(x) = -\frac{1}{2}(x+1)^2 + 1/2}$

example

79 A farmer has 1000 feet of fence to enclose a rectangular area. What are the dimensions so that the farmer encloses the maximum area.

$A = L \cdot W$ ① $2L + 2W = 1000$ ②



From ②

$2L = 1000 - 2W$

$L = 500 - W$

substitute into 1

$A = L \cdot W$

$A = (500 - W)(W)$

$\boxed{A = -W^2 + 500W}$

Quadratic equation
leading coefficient negative
so maximum value at
vertex.

$A = -(W^2 - 500W)$

$b = 500$

$\frac{b}{2} = -250$

$\left(\frac{b}{2}\right)^2 = 62500$

$\Rightarrow$ $A = -(W^2 - 500W + 62500) + 62500$

$A = -(W - 250)^2 + 62500$