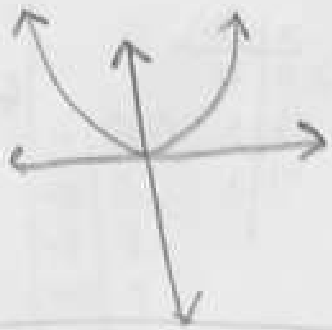
3.4 Transformations of Graphs:

Types of function we will consider:

① parabolas:

$f(x) = x^2$

Domain $\mathbb{R}$

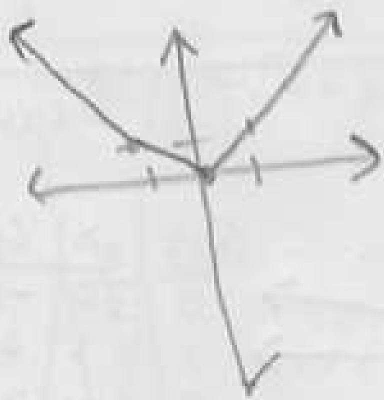


② absolute value

$f(x) = |x|$

x	$f(x)$
0	0
-1	1
1	1

Domain $\mathbb{R}$

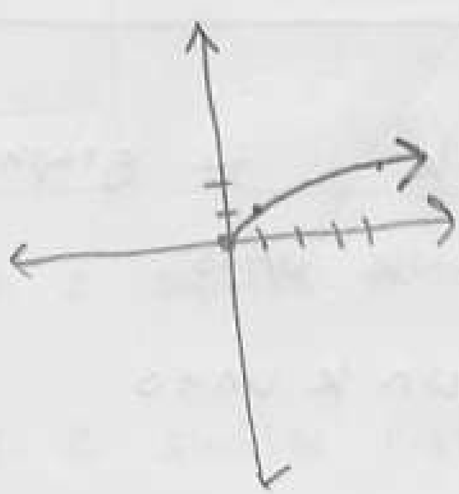


③ square root function:

$f(x) = \sqrt{x}$

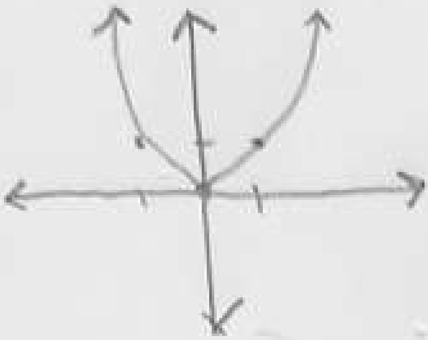
Domain $x \geq 0$

x	$f(x)$
0	0
1	1
4	2



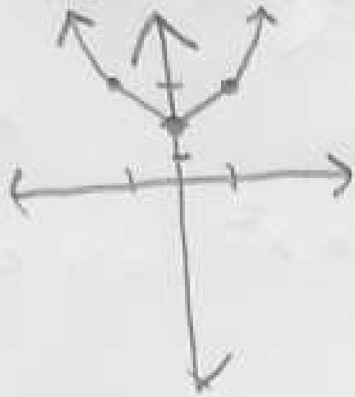
Horizontal Translations:

$f(x) = x^2$



x	f(x)
0	0
1	1
-1	1

$f(x) = x^2 + 2$

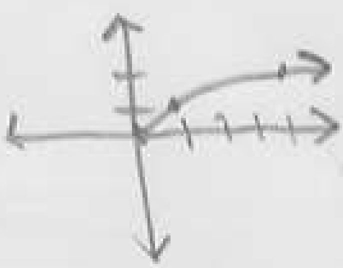


x	f(x)
0	2
1	3
-1	3

Shifted up 2 units

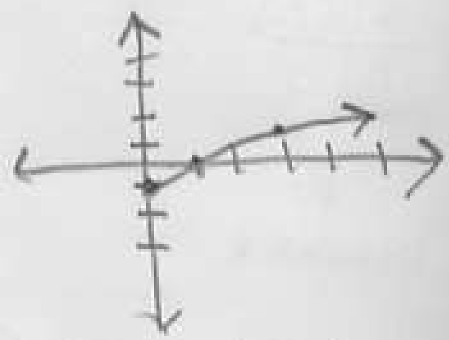
$f(x) = \sqrt{x}$

x	f(x)
0	0
1	1
4	2



$f(x) = \sqrt{x} - 1$

x	f(x)
0	-1
1	0
4	1



Shifted down 1 unit

IN GENERAL:

$f(x) + k$ is the graph of $f(x)$ shifted

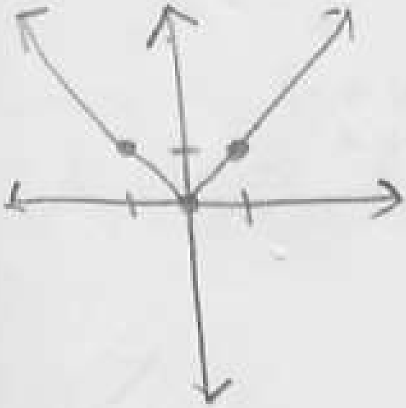
① $k > 0$ up k units

② $k < 0$ down k units

vertical transformations:

Note:
$g(x) = f(x+2)$

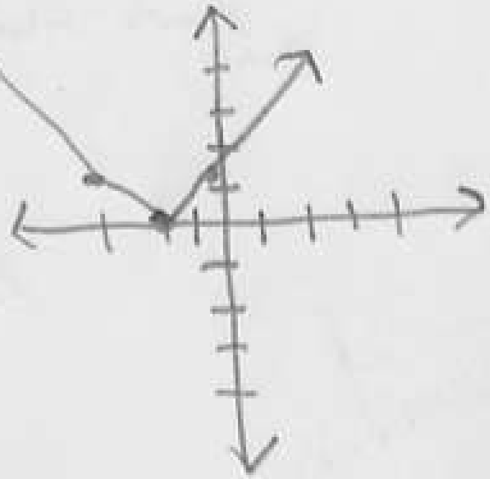
$f(x) = |x|$



x	f(x)
0	0
1	1
-1	1

$g(x) = |x+2|$

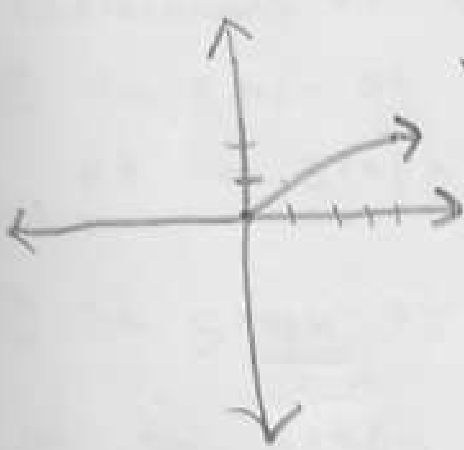
x	f(x)
-2	0
-3	1
-1	1



shifted left two units

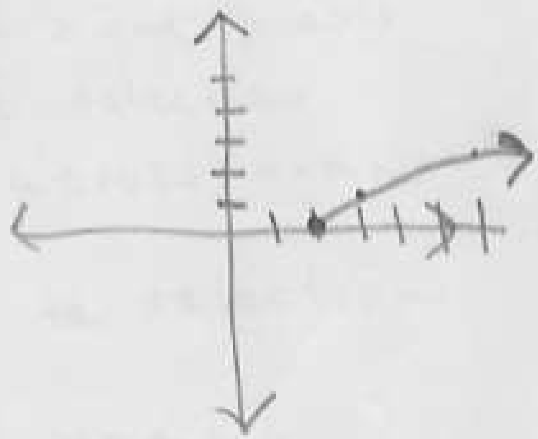
$f(x) = \sqrt{x}$

x	f(x)
0	0
1	1
4	2



$g(x) = \sqrt{x-2}$

x	f(x)
2	0
3	1
6	2



Shifted right two units

In general:

$f(x+h)$ is the graph of f shifted lefted h units

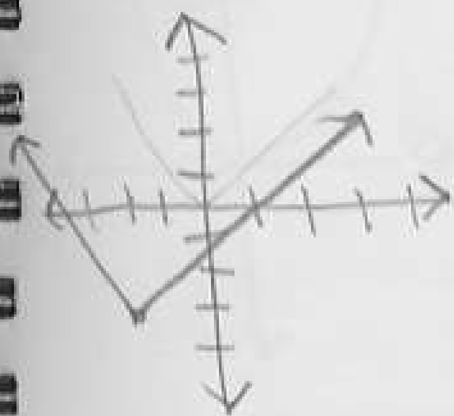
$f(x-h)$ is the graph of f shifted righ h units

Combining vertical horizontal shifts:

consider

$f(x) = |x+2| - 3$ ← shift down three units

↑ shift left 2 units



reflections of graphs across the x and y axis

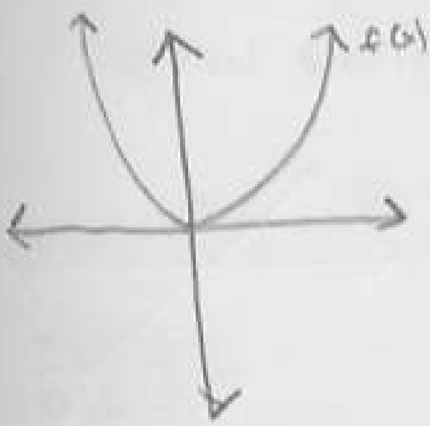
① The graph of $y = -f(x)$ is the relection of the graph of $y = f(x)$ across x-axis

② The graph of $y = f(-x)$ is the relection of the graph across the y-axis example

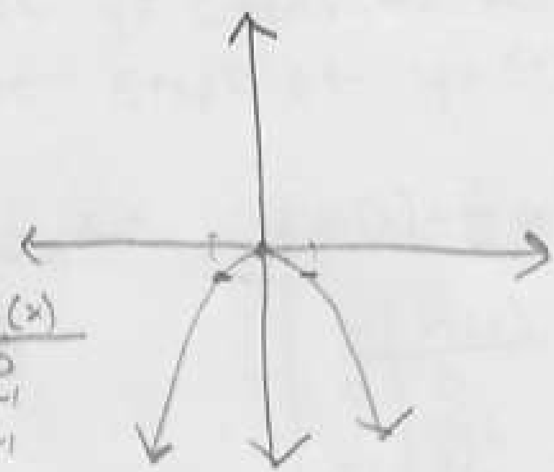
① reflection across x-axis

$f(x) = x^2$

NOTE: $g(x) = -f(x)$



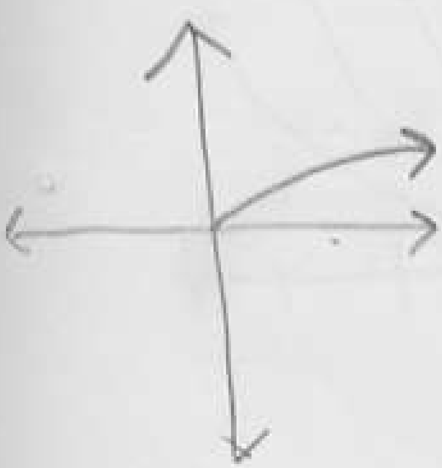
$g(x) = -x^2$



x	g(x)
0	0
1	-1
-1	-1

$f(x) = \sqrt{x}$ Domain $x \geq 0$

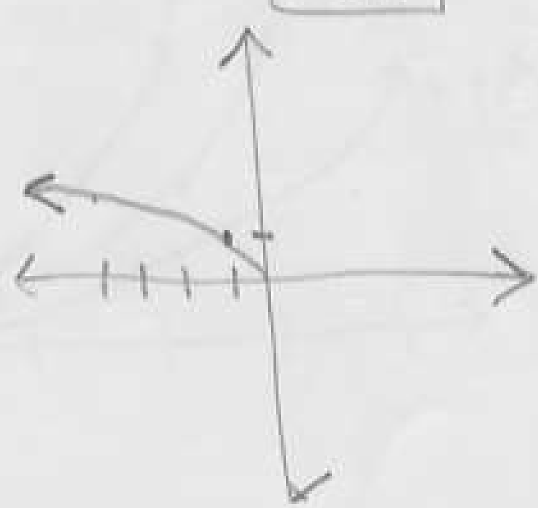
Note: $g(x) = f(-x)$



$g(x) = \sqrt{-x}$

Domain $-x \geq 0$

$\boxed{x \leq 0}$



x	g(x)
0	0
-1	1
-4	2

Vertical stretching and shrinking: Assume $c > 0$

① If $c > 1$ the graph of $y = c f(x)$ is a vertical stretching of the graph of $y = f(x)$

② If $0 < c < 1$ The graph of $y = c f(x)$ is a vertical shrinking of the graph of $y = f(x)$

$f(x) = x^2$

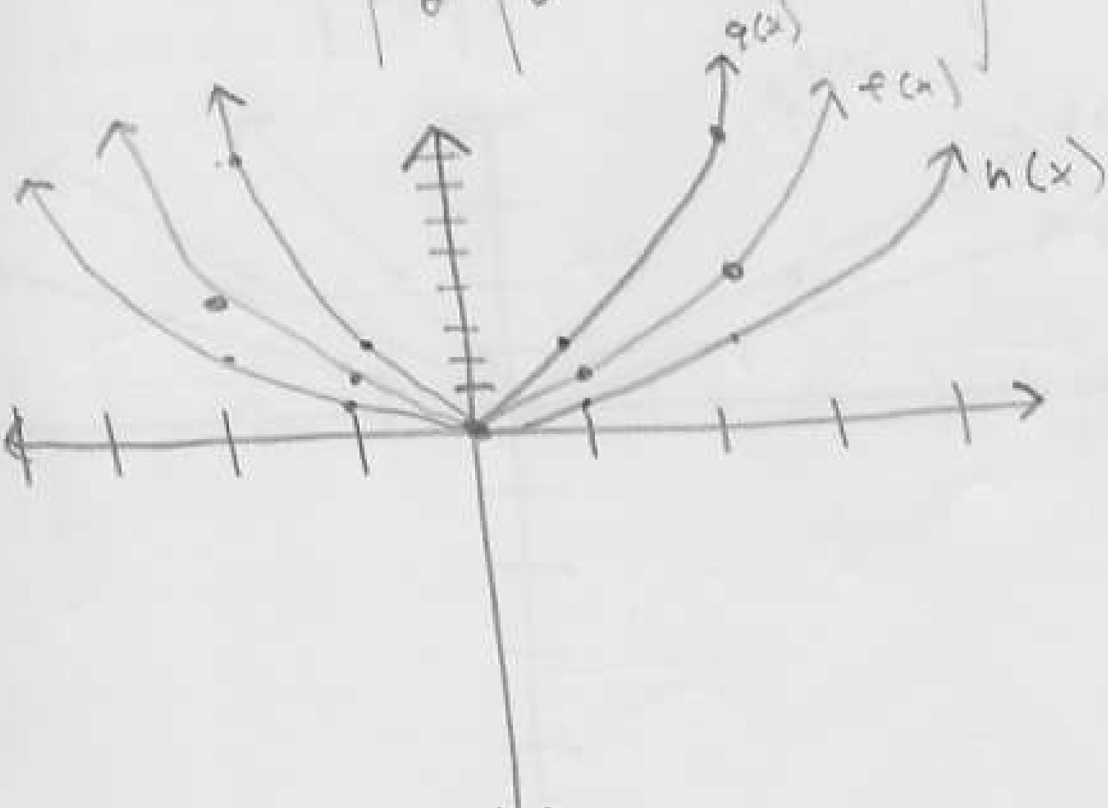
x	f(x)
0	0
1	1
-1	1
-2	4
2	4

$g(x) = 2x^2$

x	g(x)
0	0
1	2
-1	2
-2	8
2	8

$h(x) = \frac{1}{2}x^2$

x	h(x)
0	0
1	1/2
-1	1/2
-2	2
2	2



horizontal stretching and shrinking: Assume $c > 0$

① if $c > 1$ the graph of $f(cx)$ is a horizontal shrinking of the graph of $y = f(x)$

② if $0 < c < 1$, the graph of $f(cx)$ is a horizontal stretching of the graph of $f(x)$

$f(x) = |x|$

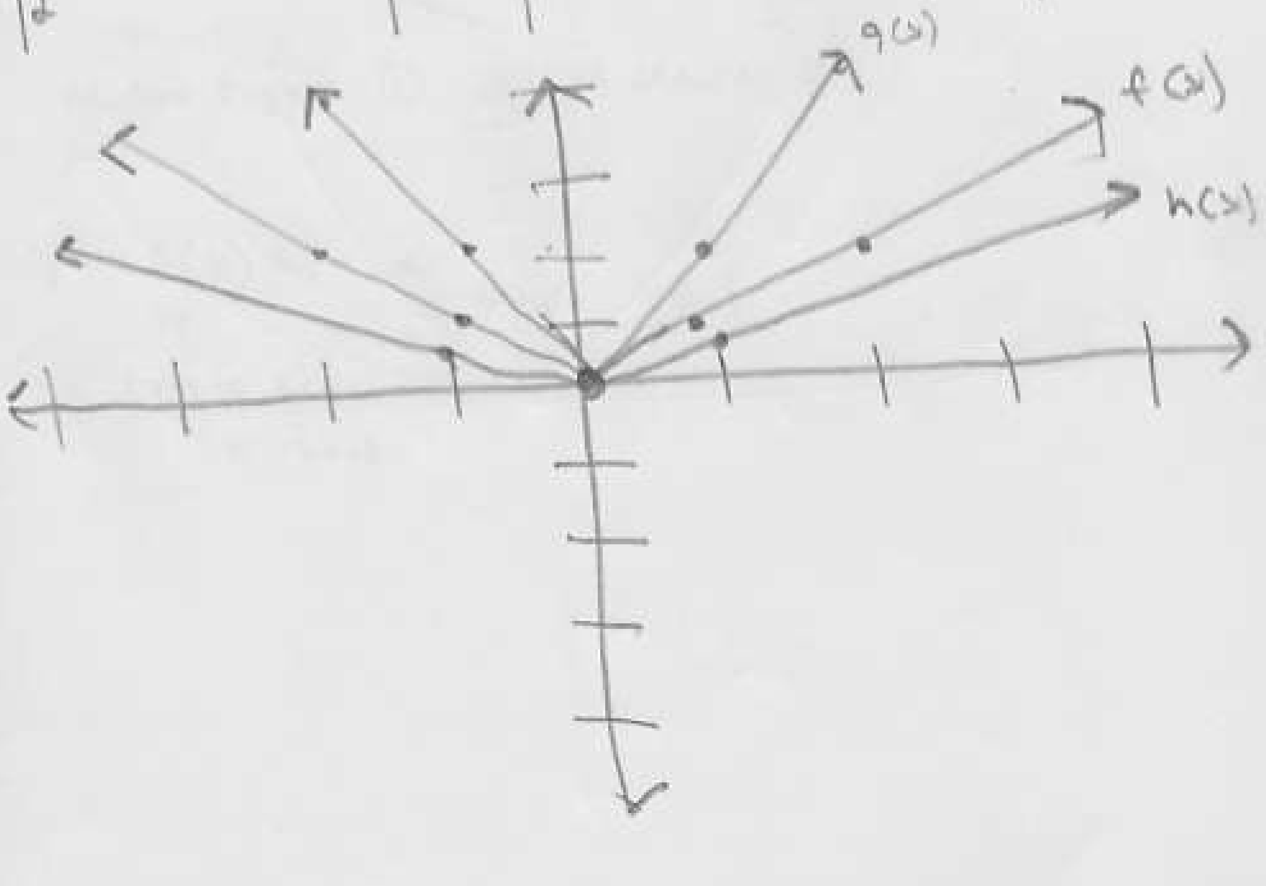
x	f(x)
0	0
1	1
-1	1
2	2
-2	2

$g(x) = |2x|$

x	g(x)
0	0
1	2
-1	2
2	4
-2	4

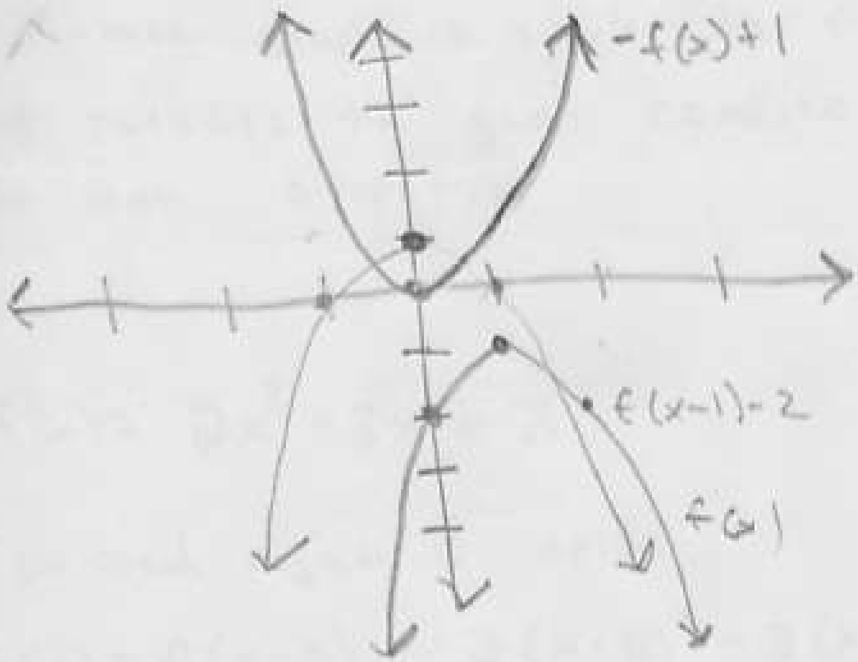
$h(x) = |\frac{1}{2}x|$

x	h(x)
0	0
1	1/2
-1	1/2
2	1
-2	1



(14)

Assume the graph represents f(x)



Graph

(a) f(x-1)-2 ← shift down 2

↑ shift right 1

(b) y = -f(x)+1 ← shift up 1

↑ rotate across x-axis

Types of questions you will be quizzed on.

(33-34)

Write a formula for a function g whose graph is similar to $f(x)$ but satisfies the given conditions. Do not simplify.

(34) $f(x) = 2x^2 - 3x + 2$

(a) shifted right 8 units

$g(x) = f(x-8) = 2(x-8)^2 - 3(x-8) + 2$

(b) shifted up 3 units

$g(x) = f(x) + 3 = (2x^2 - 3x + 2) + 3$

(c) rotated across the x axis

$g(x) = -f(x) = -[2x^2 - 3x + 2] = -2x^2 + 3x - 2$

(d) rotate across the y-axis

$g(x) = f(-x) = 2(-x)^2 - 3(-x) + 2$

$= 2x^2 + 3x + 2$

(36)

$f(x) = -3x^4 + 5x^2 - 3$

(a) shifted left 10 units and down 6 units

$g(x) = f(x+10) - 6$

$= [-3(x+10)^4 + 5(x+10)^2 - 3] - 6$

$= -3(x+10)^4 + 5(x+10)^2 - 9$

(b) shifted right 1 unit and upward 10 units

$g(x) = f(x-1) + 10$

$= -3(x-1)^4 + 5(x-1)^2 - 3 + 10$