4.1 NON-LINEAR Functions and Their Graphs

recall:

linear function can be written in the form

$$f(x) = mx + b$$

polynomial functions:

A polynomial function of degree n in the variable x can be represented by

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots a_2 x^2 + a_1 x + a_0$$

where each coefficient a_k is a real number $a_n \neq 0$, and n is a non-negative integer. The leading coefficient is a_n and the degree is n.

examples:

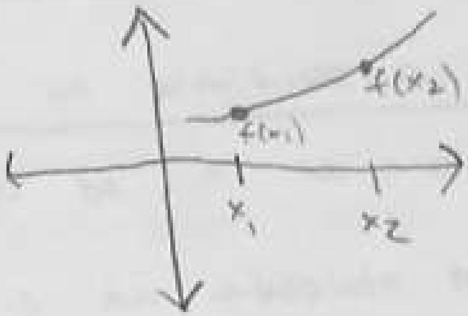
	Degree	leading Coefficient
ⓐ $f(x) = 2x + 1$	1	2
ⓑ $f(x) = 3x^2 + 2x + 1$	2	3
ⓒ $f(x) = 3x^2 + 4x^3 + 2x + 1$	3	3
ⓓ $f(x) = 4(x^0)$	0	4

polynomial with degree ≥ 2 is NON-linear

INCREASING AND DECREASING FUNCTIONS

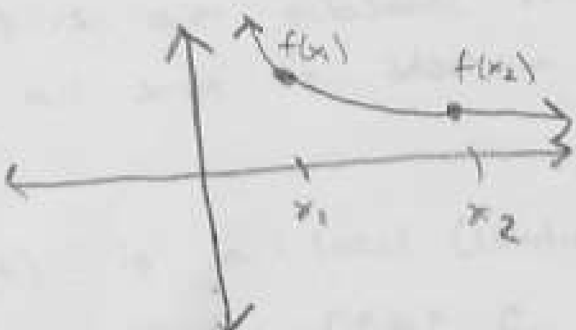
Defn. Suppose that a function f is defined over an interval I on the number line. If x_1 and x_2 are in I,

(a) f increases if whenever $x_1 < x_2$, $f(x_1) < f(x_2)$



$\underline{x_1 < x_2}$ $\quad f(x_1) < f(x_2)$

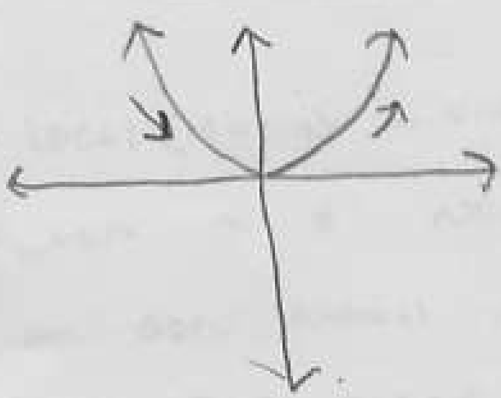
(b) f decreases on I if whenever $x_1 < x_2$ $f(x_1) > f(x_2)$



$x_1 < x_2$ $\quad f(x_1) < f(x_2)$

Consider

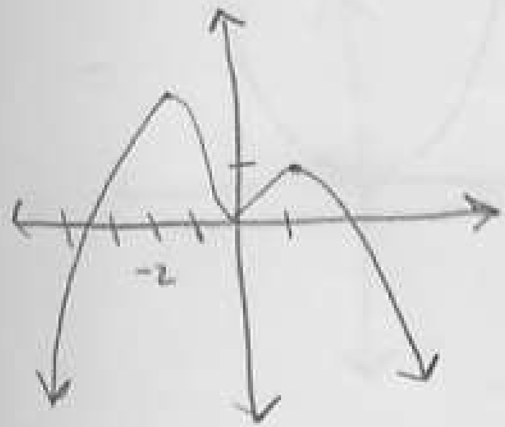
$f(x) = x^2$



Decreasing
$[-\infty, 0]$

INCREASING
$[0, \infty)$

Example: 2:



increasing= $[-\infty, -2] \cup [0, 1]$

decreasing= $[-2, 0] \cup [1, \infty]$

<u>Extrema of NONLINEAR FUNCTIONS:</u> (y value)

Let c be in the domain of f:

① f(c) is an absolute maximum (global) if $f(c) \geq f(x)$ for all x in the domain of f.

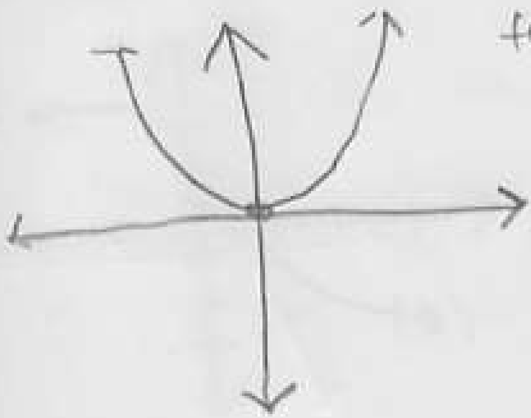
② f(c) is an absolute minimum if $f(c) \leq f(x)$ for all x in the domain of f.

③ f(c) is a local (relative) maximum if $f(c) \geq f(x)$ when x is near c.

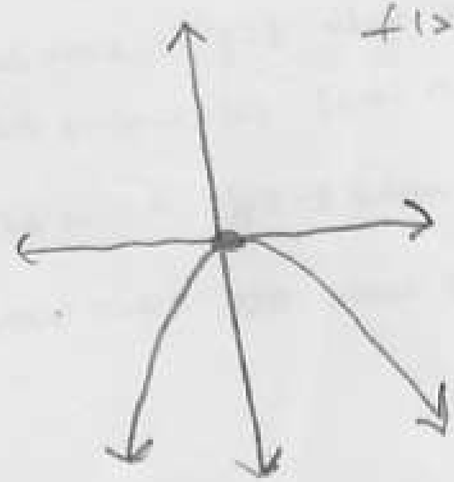
④ f(c) is a local (relative) minimum if $f(c) \leq f(x)$ when x is near c.

"near" → there is an open interval in the domain of examples:

$f(x)=x^2$



$f(x)=-x^2$



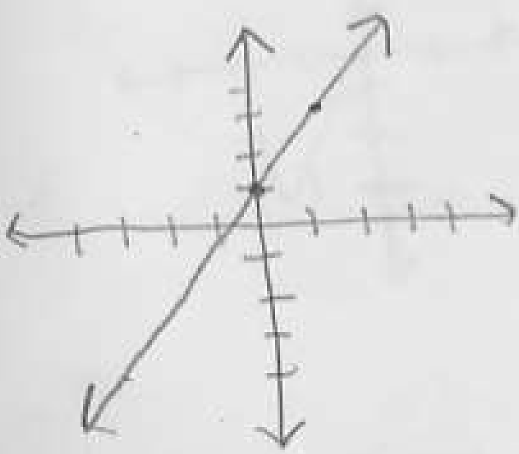
① absolute min at 0 when $x=0$;
local min at 0 when $x=0$

② Absolute Max at 0, $x=0$
local max at 0, $x=0$

☆ → parabolas have local max, absolute max] at vertex
local min, absolute min

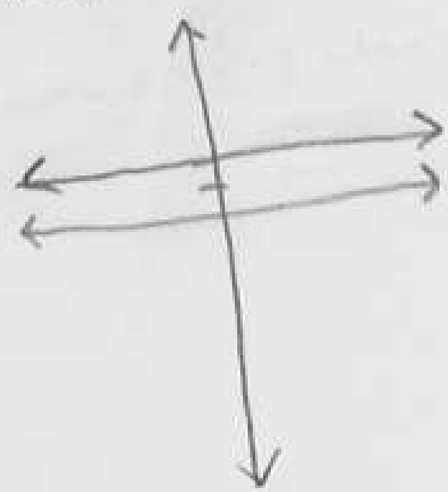
ex.

$f(x)=2x+1$



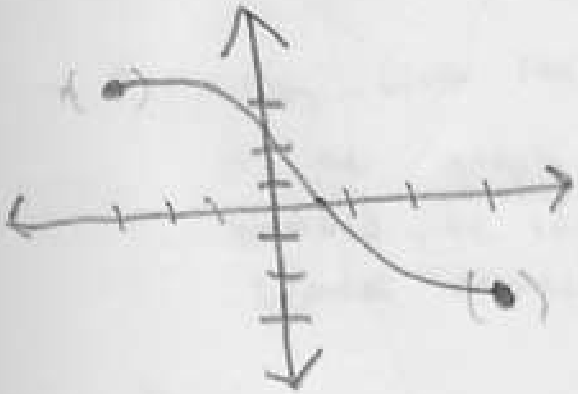
NO extrema

$f(x)=2$



Absolute max, local max
Absolute min, local min $=2$

examples:



Abs max = $y=3$ when $x=-3$

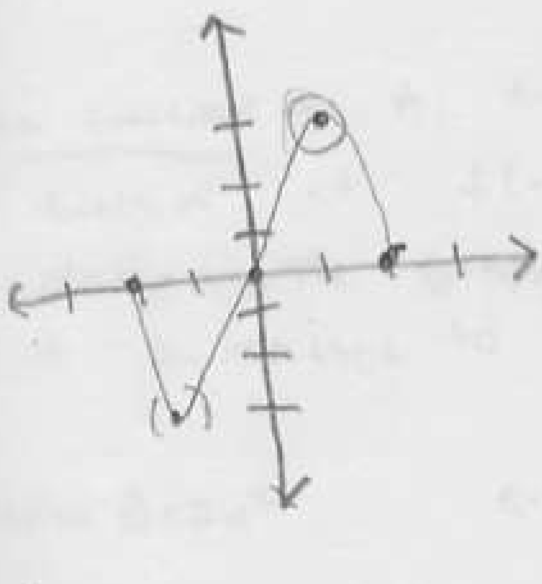
local max = No local max

Abs MIN = $y=-3$ when $x=3$

local MIN = No local MIN

HINT: Absolute extrema can occur at endpoints but local extrema cannot, → can't draw an open interval around that point

ex:



Abs max = $y=3$ when $x=1$

local MAX = $y=3$ when $x=1$

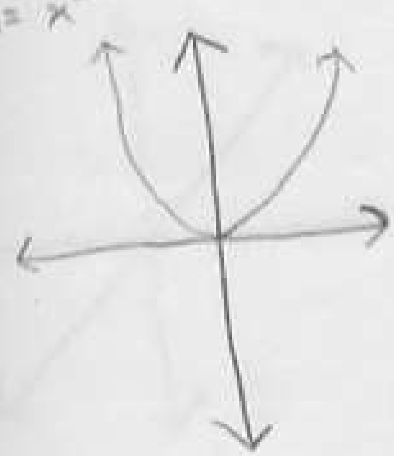
Abs MIN = $y=-3$ when $x=-1$

Local Min = $y=-3$ when $x=-1$

symmetry:

Symmetry with respect to y-axis:
if the graph were folded across the y-axis, the left and right halves would match

$f(x) = x^2$



x	$f(x)$
0	0
1	1
−1	1
2	4
−2	4

NOTICE
$f(1) = f(-1)$
$f(x) = f(-x)$

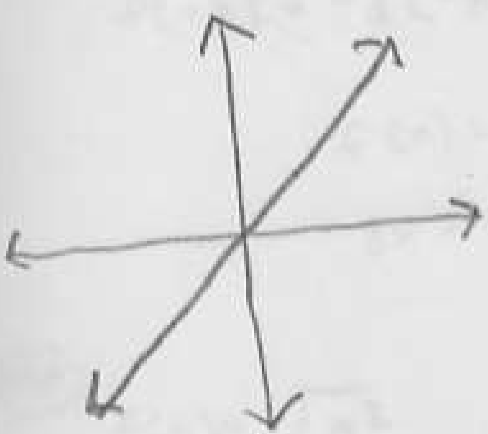
Even function → A function f is an even function if $f(-x) = f(x)$ for every x in its domain. The graph of an even function is symmetric to the y-axis

(76) $f(x) = 8 - 2x^2$ even function?

$f(-x) = 8 - 2(-x)^2 = 8 - 2x^2$

$\boxed{f(x) = f(-x) \Rightarrow \text{even function}}$

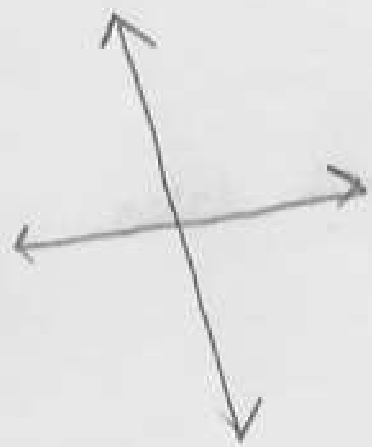
Symmetric with respect to origin: If you would spin or rotate the graph about the origin, the original graph would appear after half a turn.



$f(x) = x$

$y = x$

x	$f(x)$
1	1
2	2
−1	−1
−2	−2



NOTICE

$-f(1) = f(-1)$

$-f(x) = f(-x)$

<u>odd functions</u>: A function f is an odd function if $f(-x) = -f(x)$ for every x in its domain. (symmetric with respect to origin)

Exercises 71-90

Determine if f is even, odd, or neither

(72) $f(x) = -3x$

$f(-x) = -3(-x) = 3x$

$f(-x) = -f(x)$

$3x = -(-3x)$ ✓ odd function

(80) $f(x) = \sqrt{x^2}$

$f(-x) = \sqrt{(-x)^2} = \sqrt{x^2}$

$f(x) = f(-x)$ even

(74) $f(x) = 2x - 1$

$f(-x) = 2(-x) - 1 = -2x - 1$

$f(x) \neq -f(x)$ not odd

$f(x) \neq f(-x)$ not even

neither