4.2 Polynomial functions and Models:

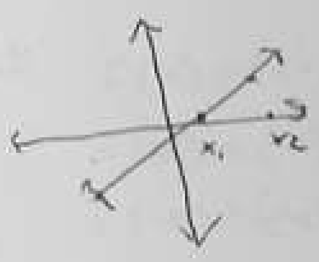
recall:

polynomial function:

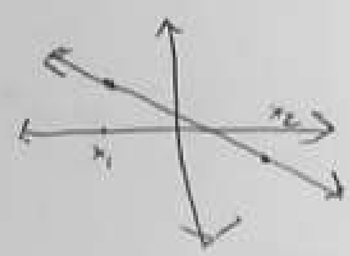
$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots a_2 x^2 + a_1 x + a_0$$

where
a_n = leading coefficient
n = degree of the polynomial.

increasing function -
where $f(x_2) > f(x_1)$ when
$x_2 > x_1$



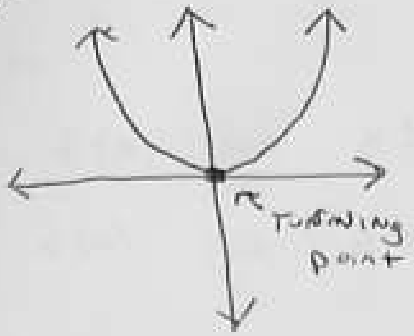
decreasing function -
where $f(x_2) < f(x_1)$
when $x_2 < x_1$



Turning point - (critical points) occurs when a graph of a function changes from increasing to decreasing or from decreasing to increasing.

example:

$f(x) = x^2$



degree = 2

leading coefficient = 1

increasing = $[-\infty, 0]$

decreasing = $[0, \infty]$

Turning point = $(0, 0)$

Local MIN = 0 when $x = 0$

Abso MIN 0 when $x = 0$

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x-intercepts and zeros

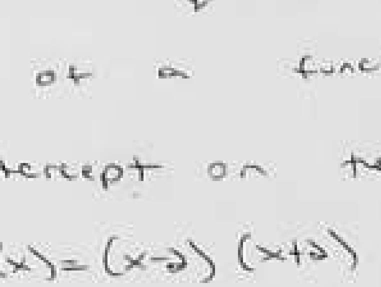
An x-intercept corresponds to an input that results in an output of 0.

consider: $f(x) = (x-2)(x+2) = x^2 - 4$

$f(2) = (0)(4) = 0$

$f(-2) = (-4)(0) = 0$

x-intercepts = $(-2, 0)$, $(2, 0)$



A zero of a function f corresponds to an x-intercept on the graph of f

$f(x) = (x-2)(x+2)$

has zero at $2, -2$ because $f(-2) = 0$, $f(2) = 0$

End behavior -

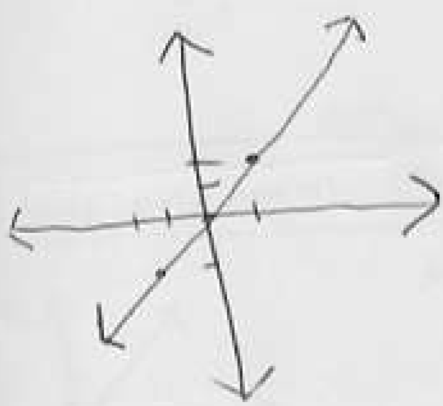
End behavior asks te question what happens to:

$f(x)$ as $x \Rightarrow \infty$

$f(x)$ as $x \Rightarrow -\infty$

excmple:

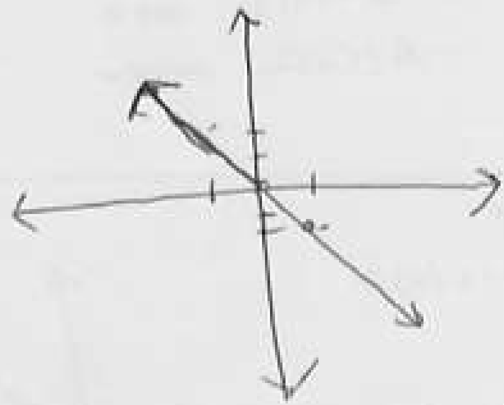
$f(x) = 2x$



as $x \Rightarrow \infty$ $f(x) \Rightarrow \infty$

$x \Rightarrow -\infty$ $f(x) \Rightarrow -\infty$

$f(x) = -2x$



as $x \Rightarrow \infty$ $f(x) \Rightarrow -\infty$

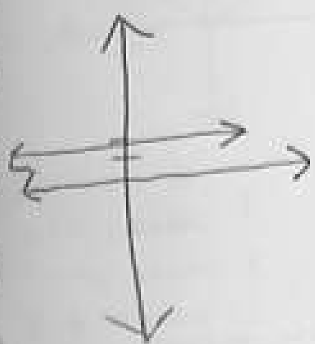
$x \Rightarrow -\infty$ $f(x) \Rightarrow \infty$

Developing trends:

(degree 0)

<u>constant polynomial functions:</u>

$f(x)=2$



degree = 0

leading coefficient = 2

turning points = none

x-intercepts = none

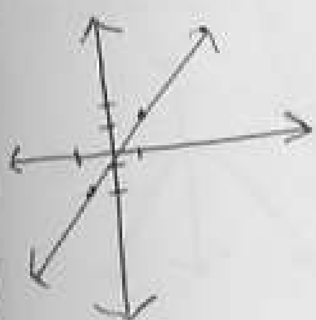
end behavior

$x \to \infty \quad f(x)=2$

$x \to -\infty \quad f(x)=2$

<u>linear polynomial functions:</u>

$f(x)=2x$



degree = 1

leading coefficient = 2

turning points = none

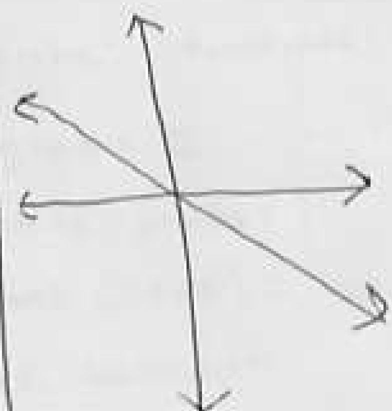
x-intercepts = 1

end-behavior:

$x \to \infty \quad f(x) \to \infty$

$x \to -\infty \quad f(x) \to -\infty$

$f(x) = -2x$



degree = 1

leading coefficient = 2

turning points = 1

x-intercepts = 1

end behavior

$x \to \infty \quad f(x) \to -\infty$

$x \to -\infty \quad f(x) \to \infty$

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linear:

x-intercepts = 1

turning points = 0

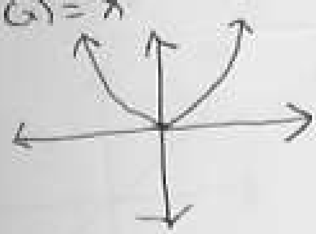
end behavior if $a_n > 0$

$f(x) \to \infty$ when $x \to \infty$

$f(x) \to -\infty$ when $x \to -\infty$

Quadratic functions

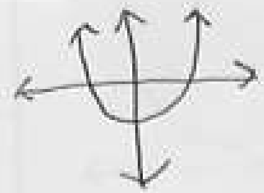
$f(x) = x^2$



degree: 2
leading coeff: 1
Turning points: 1
end behaviour:
$f(x) \to \infty$ when $x \to \infty$
~~x-int~~ $f(x) \to \infty$ when $x \to -\infty$

x-int = 1

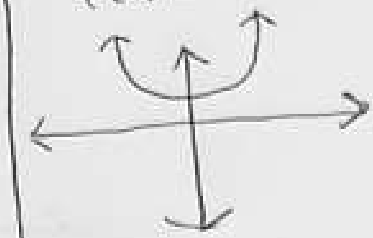
$f(x) = x^2 - 1$



degree = 2
leading coef = 1
Turning point = 1
end-behaviour =
$f(x) \to \infty$ when $x \to \infty$
$f(x) \to \infty$ when $x \to -\infty$

x-int = 2

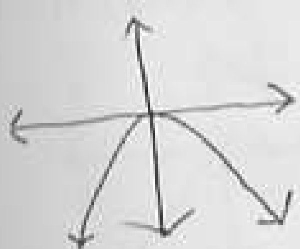
$f(x) = x^2 + 1$



degree = 2
leading = 1
T.P = 1
end-behavior
$f(x) \to \infty$, $x \to \infty$
$f(x) \to \infty$, $x \to -\infty$

x-int = 0

$f(x) = -x^2$



degree = 2
L.C = -1
T.P = 1
E.B =
$f(x) \to -\infty$ when $x \to \infty$
$f(x) \to -\infty$ when $x \to -\infty$

x-int = 1

Quadratic functions:

degree = 2

Turning points = 1

x-int (zeros) = up to 2

End behavior

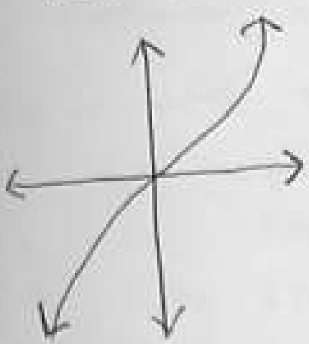
if L.C > 0
$f(x) \to \infty$ when $x \to \infty$
$f(x) \to \infty$ when $x \to -\infty$

IF L.C < 0
$f(x) \to -\infty$ when $x \to \infty$
$f(x) \to -\infty$ when $x \to -\infty$

Cubic polynomial functions:

$f(x) = x^3 = (x-0)^3$

0 has multiplicity 3



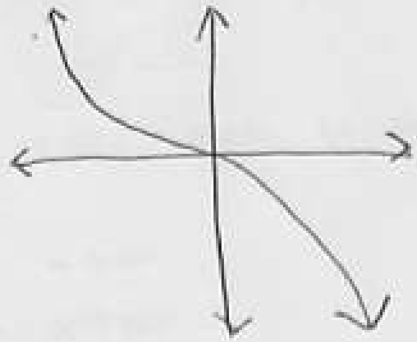
Degree = 3 L.C = 1

x-int = 1 End behavior =

$f(x) \to \infty$ when $x \to \infty$

$f(x) \to -\infty$ when $x \to -\infty$

$f(x) = -x^3$



Degree = 3 L.C = −1

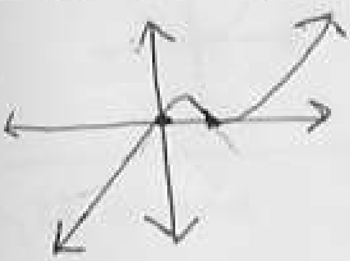
x-int = 1

End behavior =

$f(x) \to -\infty$ when $x \to \infty$

$f(x) \to \infty$ when $x \to -\infty$

$f(x) = (x-0)^2 (x-1) = x^2(x-1) = x^3 - x^2$



x-int = 2

zeros = (0, 1)

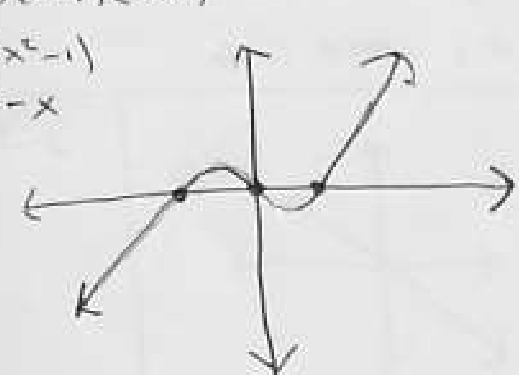
0 has multiplicity 2

$f(x) = (x-0)(x-1)(x+1)$

$= (x)(x-1)(x+1)$

$= x(x^2-1)$

$= x^3 - x$



Cubic functions:

Degree = 3

x-int = At most 3

End behavior =

L.C > 0

$f(x) \to -\infty$ when $x \to -\infty$

$f(x) \to \infty$ when $x \to \infty$

L.C < 0

$f(x) \to \infty$ when $x \to -\infty$

$f(x) \to -\infty$ when $x \to \infty$

IN GENERAL

End behaviour of polynomial functions:

Let f be a polynomial function with leading coefficient a and degree n.

① If $n \geq 2$ is even

$a > 0$ implies that f rises both to the left and right

$\Rightarrow f(x) \to \infty$ when $x \to \infty$

$f(x) \to \infty$ when $x \to -\infty$

$a < 0$ implies that f falls both to the left and right

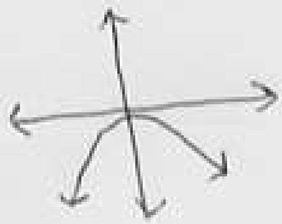
$\Rightarrow f(x) \to -\infty$ when $x \to \infty$

$f(x) \to -\infty$ $x \to -\infty$

Consider parabola

$a > 0$

$a < 0$



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② $n \geq 1$ is odd

(a) $a > 0$ implies graph falls to left, rises to right.

$f(x) \to \infty$, $x \to \infty$

$f(x) \to -\infty$, $x \to -\infty$

$a < 0$ implies graph falls to right, rises to left

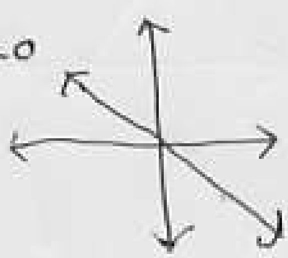
$f(x) \to -\infty$, $x \to \infty$

$f(x) \to \infty$, $x \to -\infty$

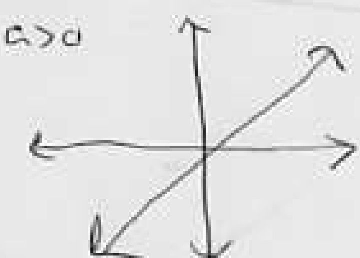
Consider line

$f(x) = ax$

$a < 0$



$a > 0$



Exercises 19-30

a) state degree and leading coefficient

b) predict end behaviour of te graph of f.

(20)

$f(x) = \frac{2}{3}x - 2$

Degree = 1

L.C = $2/3 > 0$

→ odd degree, positive L.C

$f(x) \to \infty$ when $x \to \infty$

$f(x) \to -\infty$ when $x \to -\infty$

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(22) $f(x) = 5 - \frac{1}{2}x^2$

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(21) $f(x) = 0.1x^5 - 2x^2 - 3x + 4$

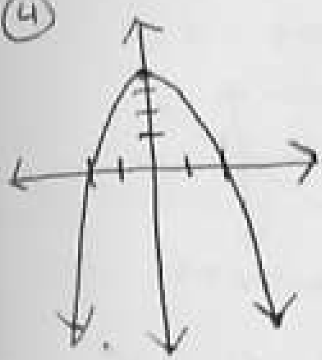
Degree, x-intercepts (zeros), Turning points

The graph of a polynomial function of degree $n \geq 1$ has at most n x-intercepts (zeros) and at most $n-1$ turning points.

ex. (3-12)

a) Determine the number of turning points
b) State whether the leading coefficient is positive or negative
c) determine the minimum degree of f:

(4)



(a) T.P = 1, x-int 2

(b) L.C =
even function:
L.C < 0

(c) minimum degree

Two inequalities

(1) # Turning points $\leq n-1$

$\Rightarrow n \geq$ # Turning points + 1 $\qquad n \geq 1+1 \Rightarrow n \geq 2$

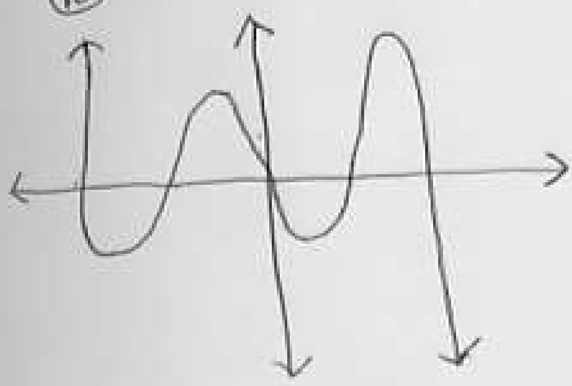
(2) # x-int $\leq n$

$\rightarrow n \geq$ # x-int. $\qquad n \geq 2$

$\boxed{\text{minimum degree } n \geq 2}$

6

10

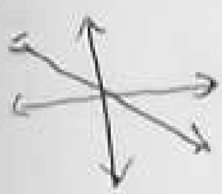


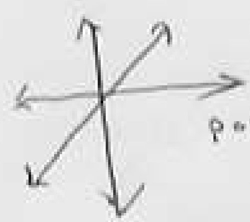
a) Turning points = 4 turning points

x-intercepts = 5 x-intercepts

b) even degree or odd degree

L.C positive or negative





positive

negative

L.C is negative

c) minimum degree

a) # Turning points $\geq n-1$

$\Rightarrow 4 \geq n-1$

$\Rightarrow n \geq 5$

b) # x-int $\leq n$

$5 \leq n$

minimum degree = 5