Dividing Polynomials

Dividend

$$\frac{38}{7} = 5 + \frac{3}{7} \Leftarrow \text{remainder}$$

Divisor Quotient

Dividend, quotient

$$\frac{X}{Y} = Q + R \Rightarrow X = YQ + YR$$

Divisor, remainder

Long division of polynomials

$$\begin{array}{r} 6x - 2 \\ x-4 \overline{)\, 6x^2 - 26x + 12} \\ -\ 6x^2 + 24x \\ \hline 0 \quad -2x + 12 \\ -\ +2x - 8 \\ \hline 4 \end{array}$$

$$= 6x - 12 + \frac{4}{x-4}$$

Dividend, quotient

$$\frac{6x^2 - 26x + 12}{x-4} = 6x - 12 + \frac{4}{x-4} \leftarrow \text{remainder}$$

$x-4 \leftarrow$ Divisor

or

$$6x^2 - 26x + 12 = (x-4)(6x-12) + 4$$

$P(x) = 3x^2 + 5x - 4$ $D(x) = x + 3$

Express in the form

$$\boxed{P(x) = D(x) \cdot Q(x) + R(x)}$$

$$\begin{array}{r} 3x - 4 + \frac{8}{x+3} \\ x+3 \overline{)\, 3x^2 + 5x - 4} \\ -\ 3x^2 - 9x \\ \hline -4x - 4 \\ +4x + 12 \\ \hline 8 \end{array}$$

$$3x^2 + 5x - 4 = (x+3)(3x-4) + 8$$

Punt's OC

(10)

$$\frac{3x^4-5x^3-20x-5}{x^2+x+3} = 3x^2-8x+1+\frac{5x-2}{x^2+x+3}$$

$$\begin{array}{r l}
 & 3x^2 - 8x + 1 \\
x^2+x+3 & \overline{)\,3x^4 - 5x^3 + 0x^2 - 20x - 5} \\
 & \underline{-3x^4 + 3x^3 - 9x^2} \\
 & -8x^3 - 9x^2 - 20x - 5 \\
 & \underline{+8x^3 + 8x^2 + 24x} \\
 & x^2 + 4x - 5 \\
 & \underline{x^2 + x + 3} \\
 & 5x - 2
\end{array}$$

Synthetic Division - can only be used when the divisor is in the form $(x-c)$

long division

$$
\begin{array}{r|l}
 & 2x^2 - x - 3 \quad r. -4 \\
\hline
x-3 & 2x^3 - 7x^2 + 0x + 5 \\
 & -2x^3 + 6x^2 \\
\hline
 & \quad -x^2 + 0x \\
 & \quad +x^2 + 3x \\
\hline
 & \qquad -3x + 5 \\
 & \qquad +3x + 9 \\
\hline
 & \qquad\quad -4
\end{array}
$$

$$\frac{2x^3-7x^2+5}{x-3} = (2x^2-x-3) + \frac{-4}{x-3}$$

Synthetic

coefficients of term

$$
\begin{array}{r|rrrr}
3 & 2 & -7 & 0 & 5 \\
\hline
 & & 6 & -3 & -9 \\
\hline
 & 2 & -1 & -3 & -4 \\
 & x^2 & x & c & r
\end{array}
= \boxed{2x^2 - x - 3 - \frac{4}{x-3}}
$$

$$\frac{x^2 - 5x + 4}{x - 1}$$

$$\begin{array}{r|rrr}
1 & 1 & -5 & 4 \\
 & & 1 & -4 \\
\hline
 & 1 & -4 & 0 \\
\hline
 & x & C & R
\end{array}
\quad = x - 4$$

$$\boxed{x^2 - 5x + 4 = (x-1)(x-4)}$$

$$\boxed{\frac{6x^4 + 10x^3 + 5x^2 + x + 1}{x + \frac{2}{3}} = 6x^3 + 6x^2 + x + \tfrac{1}{3} + \frac{1/3}{x + 2/3}}$$

$$\begin{array}{r|rrrrr}
-\frac{2}{3} & 6 & 10 & 5 & 1 & 1 \\
 & & -4 & -4 & -\frac{2}{3} & -\frac{2}{3} \\
\hline
 & 6 & 6 & 1 & \frac{1}{3} & \frac{1}{3} \\
\hline
 & x^3 & x^2 & x & C & R
\end{array}$$

Recall from Yesterday that

$$P(x) = (x-c)\,Q(x) + R$$

Dividend Divisor Quotient Remainder

IF WE PLUG IN $P(c)$

Then

$$P(c) = (c-c)\,Q(c) + R$$

$$P(c) = R$$

Remainder Theorem

IF the polynomial is divided by $x-c$, then the remainder is the value $P(c)$.

IN other words $P(c) = $ Remainder

Use Synthetic Division and Remainder Theorem
to evaluate P(c)

$P(x) = 2x^2 + 9x + 1$ $\quad c = 1/2$

$P(1/2) = 2(1/2)^2 + 9(1/2) + 1$
$= 2(1/4) + 9/2 + 1 =$
$= 1/2 + 9/2 + 1 = 6$

1

$$\begin{array}{r|rrr} 1/2 & 2 & 9 & 1 \\ & & 1 & 5 \\ \hline & 2 & 10 & 6 \\ & X & C & R \end{array}$$

2

☆

So remainder = 6

$P(x) = 5x^4 + 30x^3 - 40x^2 + 36x + 14$ $\quad c = -7$

$$\begin{array}{r|rrrrr} -7 & 5 & 30 & -40 & 36 & 14 \\ & & -35 & 35 & -35 & -497 \\ \hline & 5 & -5 & -5 & 71 & \boxed{-483} \end{array}$$

☆

$\boxed{P(-7) = 7}$

Factor Theorem

c is a zero of $P(x)$ if and only if $x-c$ is a factor of $P(x)$.

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43 What does this mean.

c is a zero of $P(x)$ if $\dfrac{P(x)}{x-c}$ has remainder

or

c is a zero of $P(x)$ if $P(c)=0$

$$\boxed{(x-c) \text{ is a factor of } P(x) \text{ if } P(c)=0}$$

44 Show that $x-c$ is a factor of $P(x)$ for the given value of c.

$$P(x)=x^3+2x^2-3x-10 \qquad c=2$$

$$\begin{array}{r|rrrr} 2 & 1 & 2 & -3 & -10 \\ & & 2 & 8 & 10 \\ \hline & 1 & 4 & 5 & 0 \\ & x^2 & x & c & R \end{array}$$

∴ From Factor Theorem $(x-2)$ is a factor

(48) Show that the given value of c are zeros of P(x) and find all other zeros

$$P(x) = 3x^4 - x^3 - 21x^2 - 11x + 6$$

$c = 1/3, -2$

$$\begin{array}{r|rrrrr} 1/3 & 3 & -1 & -21 & -11 & 6 \\ & & 1 & 0 & -7 & -6 \\ \hline & 3 & 0 & -21 & -18 & 0 \\ & x^3 & x^2 & x & c & r \end{array}$$

Then $(x - 1/3)$ is a factor

$$3x^4 - x^3 - 21x^2 - 11x = (x - 1/3)(3x^3 - 21x - 18)$$

$$\begin{array}{r|rrrr} -2 & 3 & 0 & -21 & -18 \\ & & -6 & 12 & 18 \\ \hline & 3 & -6 & -9 & 0 \\ & x^2 & x & c & r \end{array}$$

$$3x^4 - x^3 - 21x^2 - 11x = (x - 1/3)(x - 2)(3x^2 - 6x - 9)$$

$$= (x - 1/3)(x - 2)\,3(x^2 - 2x - 3)$$

$$\boxed{= (x - 1/3)(x - 2)\,3(x - 3)(x + 1)}$$

(59)

Impossible Division

Find the Remainder when $\overset{p(x)}{6x^{1000}} - 17x^{562} + 12x + 26$ is divided by $x+1$?

$P(-1) = 6(-1)^{1000} - 17(-1)^{562} + 12(-1) + 26$

$= 6 - 17 - 12 + 26$

$= 9$

Remainder $= \dfrac{9}{x+1}$

(B) Is $x-1$ a factor of $x^{567} - 3x^{400} + x^9 + 1$

Does?
$P(1) = 0$

$P(1) = (1)^{567} - 3(1)^{400} + 1^9 + 2$

$= 1 - 3 + 1 + 1$

$= 0$

So yes?