4.4 The fundamental theorem of algebra:

Complex numbers and their operations:

A complex is an expression of the standard form $a + bi$

Where a and b are Real numbers and

$i^2 = -1$ or $\sqrt{-1} = i$

examples:

$3+2i$, $3-2i$, 4

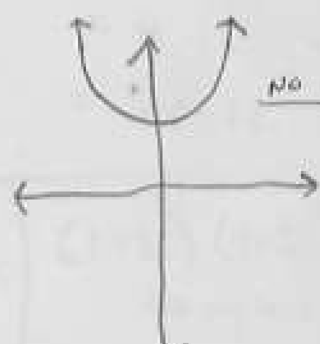
What are important about complex numbers

$$x^2 + 4 = 0$$

$$x^2 = -4$$

$$x = \pm\sqrt{-4} = \pm\sqrt{-1}\,\sqrt{4}$$

$$\boxed{x = \pm 2i}$$



No real zeros

Addition: Simply combine the real and imaginary parts
Subtraction
(combining like terms)

Addition

$(5+2i)+(3+6i) = 8+8i$ → Always in standard form

$5+(6+3i) = 11+3i$

subtraction: First need to distribute negative

$(5+2i)-(3+6i) = 5+2i-3-6i = 2-4i$

Multiplication: Always remember

$i = i$

$i^2 = i \cdot i = -1$

$i^3 = (i)(-1) = -i$

$i^4 = (-i)(i) = -(-1) = 1$

ex:

$(-2+i)(1-2i) = -2+4i+i-2i^2$

$= -2+5i+2$

$= 5i$

ex. $(2-3i)^2 = (2-3i)(2-3i)$

$= 4-6i-6i+9i^2$

$= 4-12i-9$

$= -5-12i$

$(1+2i)(1-2i)$

complex conjugates

Division: If the Denominator has a i term then multiply numerator and denominator by denominator complex conjugate.

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$$\frac{10}{1-4i}\cdot\frac{(1+4i)}{(1+4i)} = \frac{10+40i}{1-4i+4i-16i^2} = \frac{10+40i}{17} = \frac{10}{17} + \frac{40i}{17}$$

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$$\frac{(3-2i)}{(1+2i)}\cdot\frac{(1-2i)}{(1-2i)} = \frac{3-6i-2i+2i^2}{1-2i+2i-4i^2} = \frac{1-8i}{5} = \frac{1}{5}-\frac{8}{5}i$$

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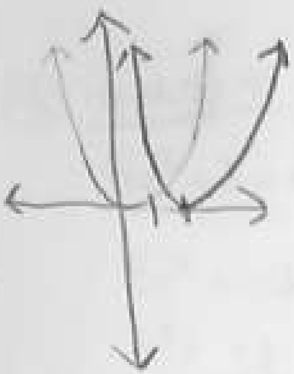
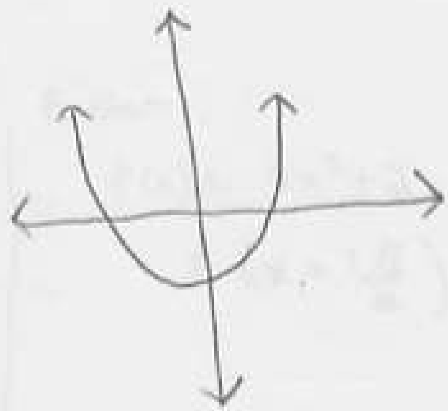
$$\frac{4-2i}{i}\cdot\frac{(i)}{(i)} = \frac{4i-2i^2}{-1} = \frac{2+4i}{-1} = -2-4i$$

want to make real

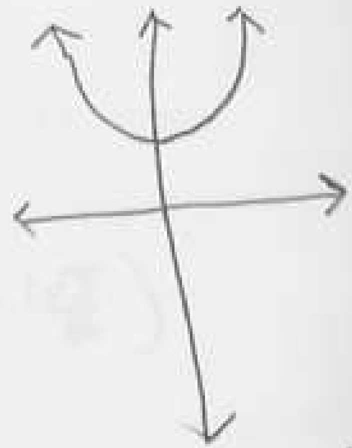
Quadratic Equations with complex solutions:



$f(x) = x^2 - 4x + 4$
$= (x-2)^2$



$f(x) = x^2 - 4$
$= (x+2)(x-2)$



$f(x) = x^2 + 4$
$= (x-2i)(x+2$

① solving for zeros

$f(x) = x^2 - 4x + 4 = 0$

4
^
-2 -2

$(x-2)^{2} = 0$

$\Rightarrow (x-2) = 0$
$(x-2) = 0$

One real zero
but zero appears twice
so
2 zero with multiplicity 2

Factors:

$f(x) = x^2 - 4 = 0$

$x^2 = 4$

$x = \pm 2$

two real zeros

± 2

$f(x) = x^2 + 4 = 0$

$x^2 = -4$

$x = \pm 2i$

Two complex zeros

$\pm 2i$

recall if k is a zero of $f(x)$

then $(x-k)$ is a factor of $f(x)$

solving quadratic equation:

(46) Find zeros

$4x^2+3=0$

$4x^2=-3$

$x^2=\frac{-3}{4}$

$x=\pm\sqrt{\frac{-3}{4}}$

$x=\pm\frac{i\sqrt{3}}{2}$

Factor

$f(x)=4x^2+3$

$=\left(x+\frac{i\sqrt{3}}{2}\right)\left(x-\frac{i\sqrt{3}}{2}\right)$

no real zeros

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$x(3x+1)=-1$

$3x^2+x=-1$

$3x^2+x+1=0$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-1\pm\sqrt{1-4(3)(1)}}{6}$

$x=\frac{-1\pm\sqrt{-11}}{6}$

Factor

$f(x)=x(3x+1)+1$

two zeros:

$x_1=\frac{-1+i\sqrt{11}}{6}$

$x_2=\frac{-1-i\sqrt{11}}{6}$

$f(x)=\left(x-\left[\frac{-1+i\sqrt{11}}{6}\right]\right)\left(x+\left[\frac{-1-i\sqrt{11}}{6}\right]\right.$

Fundamental Theorem of Algebra:

Every polynomial

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots a_1 x + a_0$$

with complex coefficients has at least one complex zero

Complex factorization Theorem:

If $p(x)$ is a polynomial of degree $n > 0$ then there exist n complex numbers such that

$$p(x) = a(x - c_1)(x \cdots c_n)$$

In other words a polynomial of degree n has n zeros.

Home work Examples:

① find all zeros of $f(x)$

② write the complete factored form of $f(x)$

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$f(x) = x^2 + 11$

① Find zeros

$x^2 + 11 = 0$

$x^2 = -11$

$x = \pm\sqrt{11}$

$x = \pm i\sqrt{11}$

② $f(x) = x^2 + 11 = (x - i\sqrt{11})^1 (x + i\sqrt{11})^1$

Check to make sure matches degree

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$f(x) = 2x^3 + 10x \quad = (x-0)^1 (x + i\sqrt{5})^1 (x + i\sqrt{5})^1$

① find zeros:

$f(x) = 2x^3 + 10x = 0$

$x(2x^2 + 10) = 0$

$x = 0 \quad$ or $\quad 2x^2 + 10 = 0$

$x \pm -i\sqrt{5}$

$f(x) = x^4 + 2x^2 + 1$ <u>4 zeros</u>

① let $u = x^2$

Then

$f(u) = u^2 + 2u + 1 = 0$

$= (u+1)^2 = 0$

$\downarrow$

$(u+1)^2 = 0$ ← multiplicity 2

$u+1=0$ or $u+1=0$

but $u = x^2$

$x^2+1=0$ or $x^2+1=0$

$x^2=-1$ or $x^2=-1$

$x = \pm i$ or $x = \pm i$

$$\boxed{f(x) = (x-i)^2 (x+i)^2}$$