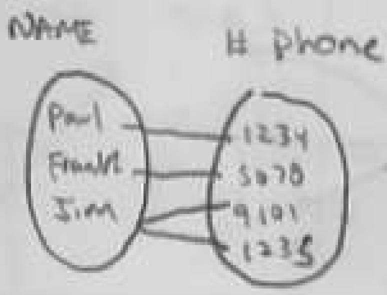
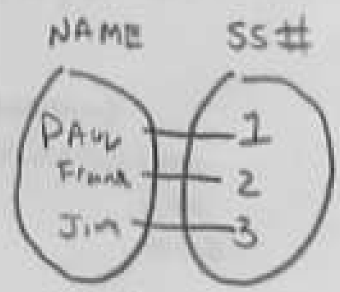
Inverse functions: 5.2

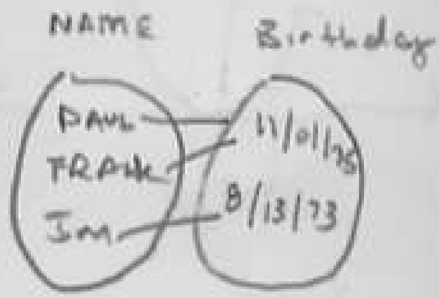
One - to - one functions:



NOT A Function



Function:
ONE-to-ONE

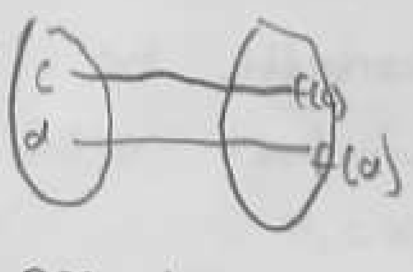


FUNCTION:
NOT ONE TO-ONE

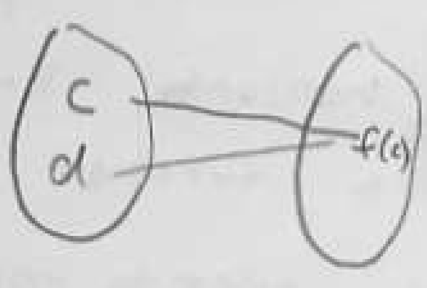
Defn: A function is one to one if, for elements c and d in the domain f

$$c \neq d \Rightarrow f(c) \neq f(d)$$

That is different inputs result in different outputs



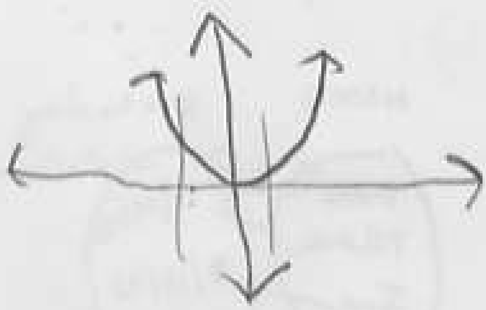
one - to - one



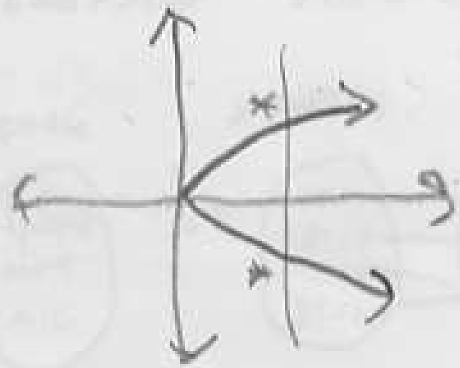
Not - one - to - ONE

Graphically: How to tell if a function is one-to-one:

Functions: vertical line test:



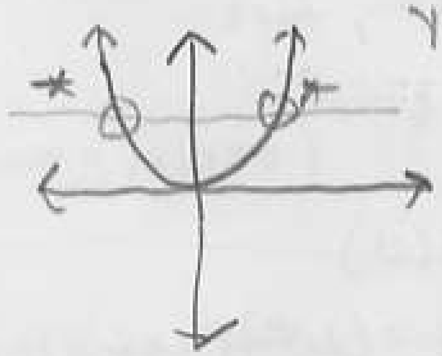
Function



Not a Function.

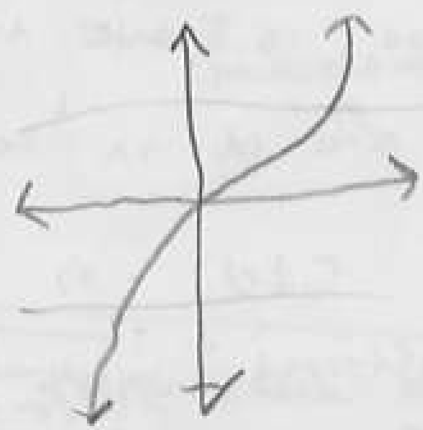
One-to-one: Horizontal line test

$y = x^2$



Not one to one

$y = x^3$



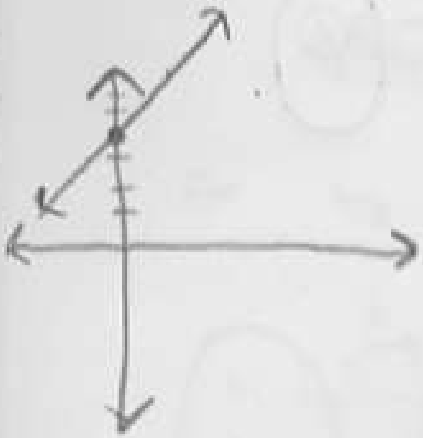
One-to-one

If every horizontal line intersects the graph of a function f at most once then f is one-to-one.

Algebraically showing that a function is one-to-one:

Building a mathematical proof:

Show that $f(x) = 3x + 4$ is one-to-one.



passes horizontal line test

① suppose there are numbers x_1 and x_2 such that $f(x_1) = f(x_2)$. <u>IF one-to-one then $x_1 = x_2$</u>.

Assume $f(x_1) = f(x_2)$

$$3x_1 + 4 = 3x_2 + 4$$
$$-4 \qquad\qquad -4$$

$$\frac{3x_1}{3} = \frac{3x_2}{3}$$

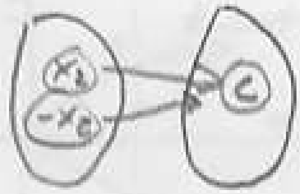
$$x_1 = x_2$$

So function is one-to-one.

Show algebraically that $f(x) = x^2$ is not one-to-one

Assume $f(x_1) = f(x_2) = c$

Then



$$x_1^2 = x_2^2$$

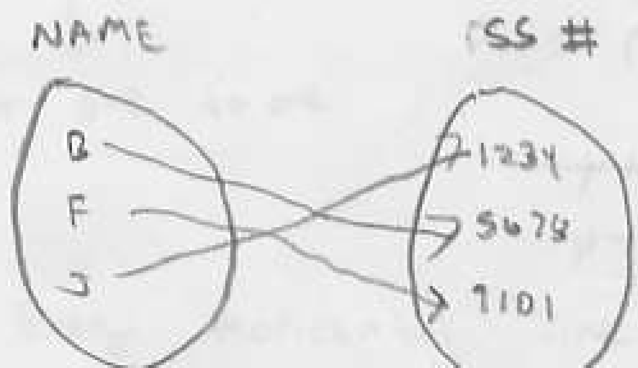
$$\sqrt{x_1^2} = \pm\sqrt{x_2^2}$$

$$x_1 = \pm x_2$$

$x_1 = x_2$ and $x_1 = -x_2$

So not one-to-one

What's important about 1-1 functions:



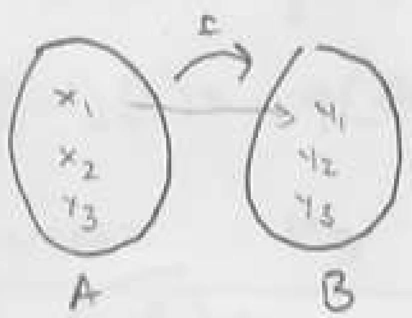
Given A SS# you can track back the persons NAME.

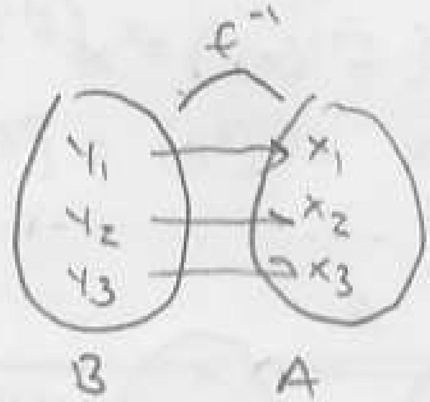
The birthday example from before we can not track back the name!

Defn: <u>Inverse function</u>: Let f be a one-to-one function with domain A and range B. Then its inverse function f^{-1} has domain B and range A and is defined by

$$f^{-1}(y) = x \iff f(x) = y$$

for any y in B.





$f(x_1) = y_1$ $\qquad$ $f^{-1}(y_1) = x_1$

$f(x_2) = y_2$ $\qquad$ $f^{-1}(y_2) = x_2$

$f(x_3) = y_3$ $\qquad$ $f^{-1}(y_3) = x_3$

IMP

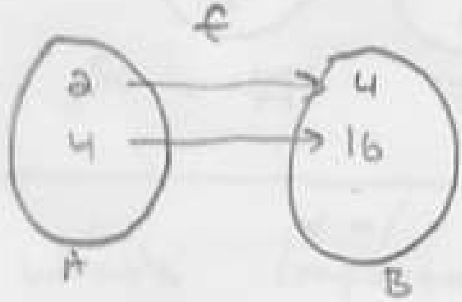
Property of INVERSE functions:

Let f be a 1-1 function with domain A and RANGE B. The inverse function f^{-1} Satisfies the following cancellation properties:

$(f^{-1} \circ f)(x) = f^{-1}(f(x)) = x$ for every x in A

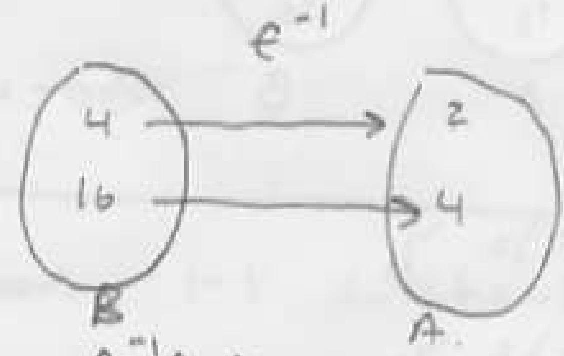
$(f \circ f^{-1})(x) = f(f^{-1}(x)) = x$ for every x in B

example



$f(2) = 4$

$f(4) = 16$



$f^{-1}(4) = 2$

$f^{-1}(16) = 4$

$f^{-1}(f(4)) = f^{-1}(16) = 4$

$f(f^{-1}(4)) = f(2) = 4$

Verifying that two function are inverses.

① $(f \circ g)(x) = x$

② $(g \circ f)(x) = x$

Let $f(x) = x^3$ $\quad g(x) = x^{1/3}$

$$(f \circ g)(x) = f(g(x)) = f(x^{1/3}) = (x^{1/3})^3 = x$$

$$(g \circ f)(x) = g(f(x)) = g(x^3) = (x^3)^{1/3} = x$$

So inverses.

Verify $f(x)$ and $g(x)$ are inverses:

$f(x) = 2x - 5$ $\quad g(x) = \frac{x+5}{2}$

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{x+5}{2}\right) = 2\left(\frac{x+5}{2}\right) - 5 =$$

$$= x + 5 - 5$$

$$= x$$

$$(g \circ f)(x) = g(f(x)) = g(2x-5) = \frac{(2x-5)+5}{2} = \frac{2x}{2} = x$$

SO INVERSES.

How to find the inverse of a one-to-one function.

1. write $y = f(x)$
2. Solve this equation for x in terms of y
3. interchange x and y.
4. write y as $f^{-1}(x)$

---

ex.

Find inverse of $f(x) = 3x - 2$

1. $\boxed{y = 3x - 2}$

2. $3x = y + 2$

   $\boxed{x = \frac{y+2}{3}}$

3. $y = \frac{x+2}{3}$

4. $f^{-1}(x) = \frac{x+2}{3}$

---

check

1. $(f \circ f^{-1})(x) = f(f^{-1}(x)) = f\left(\frac{x+2}{3}\right) = 3\left(\frac{x+2}{3}\right) - 2 = x$

2. $(f^{-1} \circ f)(x) = f^{-1}(f(x)) = f^{-1}(3x-2) = \frac{3x-2+2}{3} = \frac{3x}{3} = x$ ✓

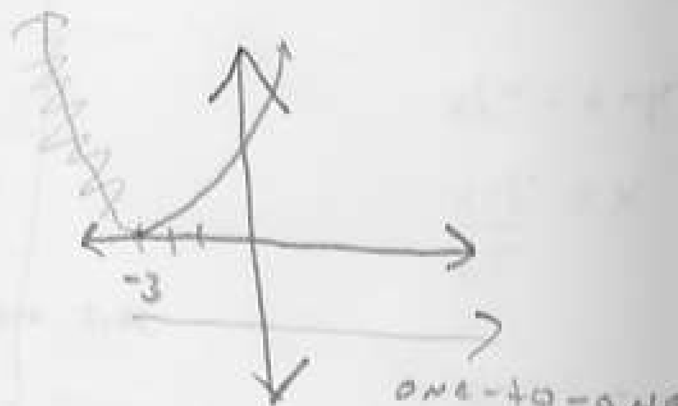
Exercise (71-84)

Find a symbolic representation for $f^{-1}(x)$

identify the domain and range of $f^{-1}(x)$

verify that f and f^{-1} are inverses.

$f(x) = (x+3)^2 \quad x \geq -3$



#72

① $y = (x+3)^2$

② $y = (x+3)^2$

$\pm\sqrt{y} = (x+3)$ but x

$x = -3 \pm \sqrt{y}$ but $x \geq -3$ so don't need to worry about $-\sqrt{y}$

$x = -3 + \sqrt{y}$

③ $y = -3 + \sqrt{x}$

④ $f^{-1}(x) = -3 + \sqrt{x}$

(29)

$f(x) = 6 - 7x$

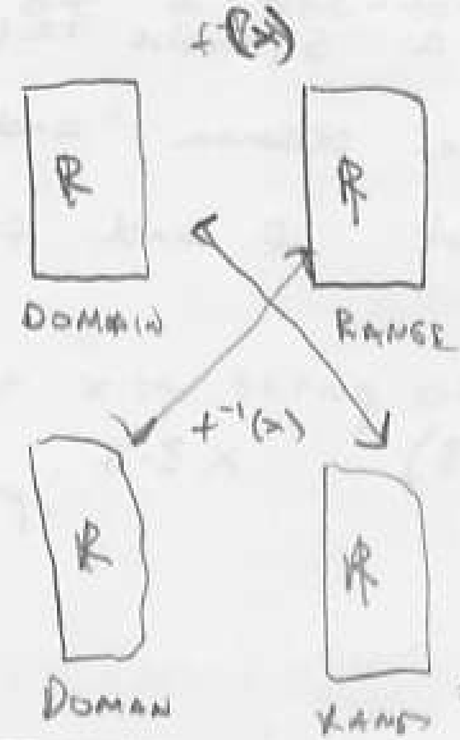
① $y = 6 - 7x$

② $y - 6 = -7x$

$x = \frac{y-6}{-7}$

③ $y = \frac{x-6}{-7}$

④ $\boxed{f^{-1}(x) = \frac{x-6}{-7}}$



check:

$f(f^{-1}(x)) = f\left(\frac{x-6}{-7}\right) = 6 - 7\left(\frac{x-6}{-7}\right) = 6 + x - 6 = x$

$f^{-1}(f(x)) = f^{-1}(6-7x) = \frac{(6-7x)-6}{-7} = \frac{-7x}{-7} = x$

(56)

$$f(x) = \frac{1-x}{3x+1}$$

① $y = \frac{1-x}{3x+1}$

②

$(3x+1)(y) = 1-x$

$3xy + y = 1-x$

$3xy + x = 1-y$ → bring x's to one side

$x(3y+1) = 1-y$ → factor

$$x = \frac{1-y}{3y+1}$$

③ $y = \frac{1-x}{3x+1}$

④ $f^{-1}(x) = \frac{1-x}{3x+1}$

---

check

$$f(f^{-1}(x)) = f\left(\frac{1-x}{3x+1}\right) = \frac{1-\left(\frac{1-x}{3x+1}\right)}{3\left(\frac{1-x}{3x+1}\right)+1}$$

$$\frac{1-\left(\frac{1-x}{3x+1}\right)}{3\left(\frac{1-x}{3x+1}\right)+1} = \frac{\frac{3x+1}{3x+1} - \frac{1-x}{3x+1}}{\frac{3(1-x)}{3x+1} + \frac{3x+1}{3x+1}}$$

$$= \frac{\frac{3x+1-(1-x)}{3x+1}}{\frac{3(1-x)+3x+1}{3x+1}}$$

$$= \frac{\frac{4x}{3x+1}}{\frac{4}{3x+1}} = \left(\frac{4x}{\cancel{3x+1}}\right)\left(\frac{\cancel{3x+1}}{4}\right) = \frac{4x}{4} = x$$