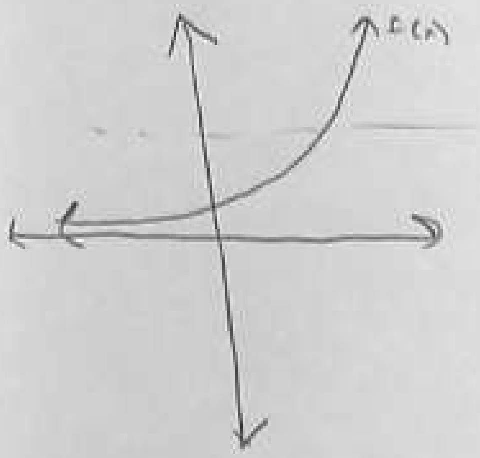
5.4

Logarithmic functions: (PART 1) $\boxed{\text{HW upto 39}}$

Let's consider the exponential function

$f(x) = a^x$ where $a > 1$



We can notice that $f(x)$ is a 1-1 function. This implies that $f(x)$ has an inverse. The inverse of this function is called a logarithmic function.

Defn: Logarithmic function:

Let a be a positive number with $a \neq 1$. The logarithmic function with base a, denoted by $\log_a$, is defined by

$$\log_a x = y \iff a^y = x$$

↑ logarithmic form ↑ exponential form

☆ $\log_a x$ is the exponent to which a must be raised to get x.

When working with logarithmic functions it is extremely useful to be comfortable with switching between logarithmic and exponential forms:

$$\boxed{\begin{array}{l} \text{logarithmic form:} \\ \qquad \log_a x = ? \\ \qquad a \text{ to what power} = x \end{array}}$$

useful way to think about logarithms

example: converting from logarithmic to exponential form

	log		exp
ⓐ	$\log_{10} 1000 = 3$	$\Rightarrow$	$10^3 = 1000$
ⓑ	$\log_2 32 = 5$	$\Rightarrow$	$2^5 = 32$
ⓒ	$\log_{16} 4 = \frac{1}{2}$	$\Rightarrow$	$16^{1/2} = 4$

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	exp		log
ⓐ	$2^3 = 8$	$\Rightarrow$	$\log_2 8 = 3$
ⓑ	$10^5 = 100{,}000$	$\Rightarrow$	$\log_{10} 100{,}000 = 5$
	$25^{1/2} = 5$	$\Rightarrow$	$\log_{25} 5 = 1/2$

solving simple logarithmic functions: (rewrite in exponential form)

ⓐ $\log_2 x = 5$ $\Rightarrow$ $2^5 = x$

$\Rightarrow$ $\boxed{x = 32}$

ⓑ $\log_5 x = 0$

$5^0 = x$

$1 = x$

ⓒ $\log_4 16 \Rightarrow$ $\boxed{\text{4 to what power} = 16}$

① set equal to y

$\log_4 16 = y$

② rewrite in expontential form

$4^y = 16$

③ make bases equal (one way to solve)

$4^y = 4^2$

$\boxed{\Rightarrow y = 2}$

ⓓ $\log_5 5^4$ $\Rightarrow$ 5 to what power $= 5^4$

① set equal to y:

$y = \log_5 5^4$

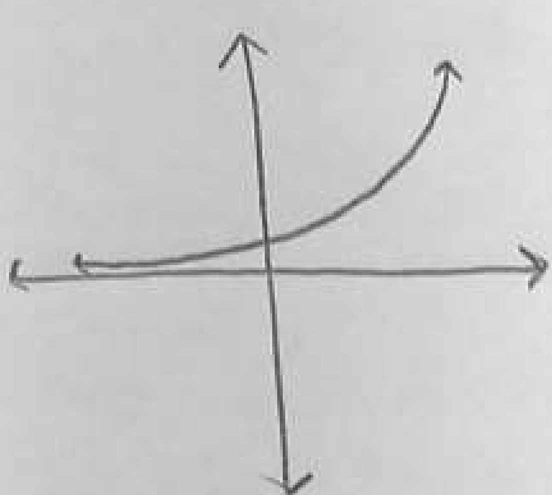
$5^y = 5^4$

$\boxed{y = 4}$

Properties of Logarithms:

① Domain / RANGE:

Recall $f(x) = a^x$

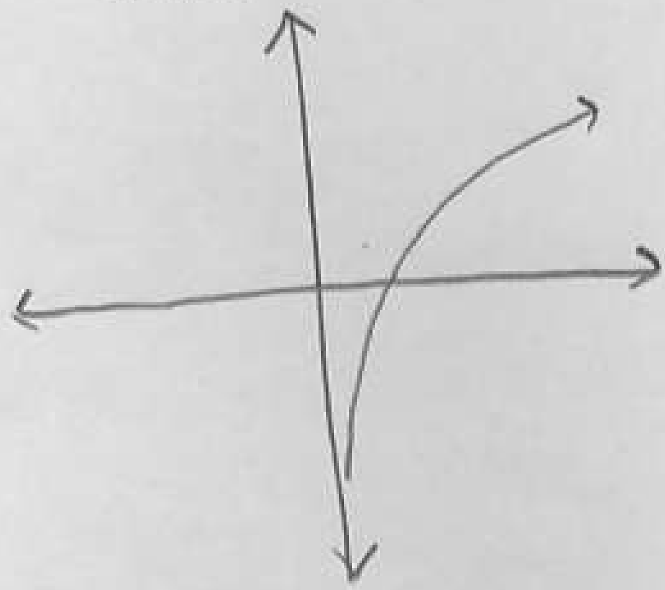


Domain = $\mathbb{R}$

RANGE = $(0, \infty)$

Since $g(x) = \log_a x$

Then domain and RANGE switch



Domain = $(0, \infty)$

RANGE = $\mathbb{R}$

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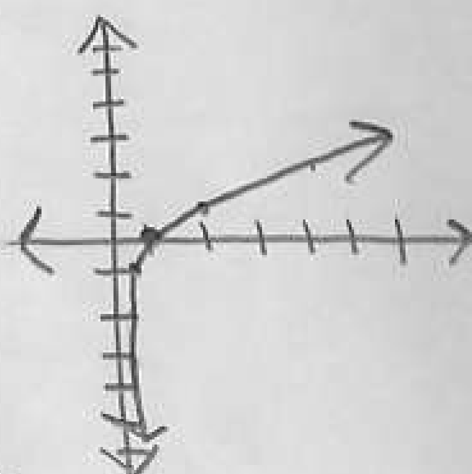
example graph $g(x) = \log_2 x$

x	$g(x)$			
1	0	$\Rightarrow \log_2 1 = y$	$2^y = 1$	$y = 0$
2	1	$\log_2 2 = y$	$2^y = 2$	$y = 1$
$\frac{1}{2}$	-1	$\log_2 \frac{1}{2} = y$	$2^y = \frac{1}{2}$	$y = -1$
4	2	$\log_2 4 = y$	$2^y = 4$	$y = 2$
$\frac{1}{4}$	-2	$\log_2 {}^1/_4 = y$	$2^y = \frac{1}{4}$	$y = -2$



Algebraic properties:

① $\log_a 1 = 0$ because $a^0 = 1$

example

$$\log_5 1 = 0$$

② $\log_a a = 1$ because $a^1 = a$

example

$$\log_\pi \pi = 1$$

INVERSE PROPERTIES:

③ $\log_a a^x = x$ because $a^x = a^x$

example:

$$\log_5 5^4 = 4$$

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④ $a^{\log_a x} = x$ because $\log_a x$ is the power to which a must be raised to get x.

ex. If $\log_a x = y$

Then $a^y = x$

So $a^y = x$

Think about this in terms of inverses

$f(x) = a^x$ $\qquad$ $g(x) = \log_a x$

if inverses then

ⓐ $f(g(x)) = x$

$\Rightarrow f(\log_a x) = x$

$\Rightarrow \boxed{a^{\log_a x} = x}$

ⓑ $g(f(x)) = x$

$\Rightarrow g(a^x) = x$

$\boxed{\log_a a^x = x}$

problems using properties: $f(x) = \log_a(x)$

① $\log_5(-\pi)$ undefined because x must be greater than 0.

② $\log_6 6^2 = 2$

3^{1}

③ $5^{\log_5 3} = 3$

④ $\log_4 1 = 0$

⑤ $\log_\pi \pi = 1$

recall:

$10^4 = 10000$

$10^3 = 1000$

$10^2 = 100$

$10^1 = 10$

$10^0 = 1$

$10^{-1} = .01$
1 zero

$10^{-2} = .\underbrace{00}_{\text{2 zero}}1$

$10^{-3} = .0001$

Why do we have special common log:

because powers of 10 are so easy to deal with!

Specially defined logarithms:

① common logarithm:

The logarithm with base 10 is called the common logarithm and is denoted by ommiting the base

$$\log x = \log_{10} x$$

$$\boxed{y = \log x \Leftrightarrow 10^y = x}$$

Properties:

$\log 1 = 0 \quad \Rightarrow \quad 10^0 = 1$

$\log 10 = 1 \quad\quad 10^1 = 10$

inverse properties

$10^{\log x} = x$ $\quad y = \log x \Rightarrow 10^y = x$

$\Rightarrow 10^y = x$

$\log 10^x = x \quad \Rightarrow \quad 10^x = 10^x$

IN your calulartor you will have a $\boxed{\log}$ button examples:

ⓐ $\log 100 = \boxed{2}$

$y = \log 100$ ⇒ set equal to y

$10^y = 100$ ⇒ convert to exponetial form

$\boxed{10^y = 10^2}$ ⇒ same bas.

$\boxed{y = 2}$

ⓑ $\log 0.01 \boxed{= -2}$

$y = \log 0.01$ → Set equal to y

$10^y = .01$

$10^y = 10^{-2}$

$y = -2$

ⓒ $\log 0.1 - \log 1000 = z - y = -2 - 3 = -5$

Do them separately:

$z = \log 0.1$

$10^z = .01$

$\boxed{z = -2}$

$y = \log 1000$

$10^y = 1000$

$\boxed{y = 3}$

Natural logarithms:

The logarithm with base e is called the natural logarithm and is denoted by ln:

$$\ln x = \log_e x$$

$$\ln x = y \iff e^y = x$$

Properties

① $\ln 1 = 0 \Rightarrow e^0 = 1$

② $\ln e = 1 \Rightarrow e^1 = e$

③ $\ln e^x = x \Rightarrow e^x = e^x$

④ $e^{\ln x} = x$

examples:

① $\ln e^4 = 4$

② $e^{\ln 5x} = 5x$

③ $\ln 1 = 0$

④ $\ln(-4) =$ undefined

pluggins into calculator:

(a) $\log 500$

(b) $\ln 500$

(c) $\ln(e^2)$

(d) $\log(e^2)$