5.5 Properties of logarithms:

Review:

① $\log_a 1 = 0 \Rightarrow a^0 = 1$

② $\log_a a = 1 \Rightarrow a^1 = a$

③ $\log_a a^x = x \Rightarrow a^x = a^x$

④ $a^{\log_a x} = x$

more properties:

⑤ $\log_a m + \log_a n = \log_a (mn)$

⑥ $\log_a m - \log_a n = \log_a (m/n)$

⑦ $\log_a (m^r) = r \log_a m$

examples:

ⓐ $\ln 5 + \ln 6 = \ln(5 \cdot 6) = \ln(30)$

From property ⑤

ⓑ $\ln 5 - \ln 6 = \ln(5/6)$

property ③

ⓒ $\ln e^2 = 2 \ln e = 2(1)$

property 7   property ②

Proofs:

Let $\log_a A = u$ and $\log_a B = V$

$\Rightarrow a^u = A$ and $a^V = B$

① prove: $\log_a (AB) = \log_a(A) + \log_a(B)$

$$\log_a(AB) = \log_a(a^u a^v) = \log_a(a^{u+v})$$
$$= u + v$$
$$= \log_a A + \log_a B$$

② $\log_a(A/B) = \log_a(A) - \log_a(B)$

$$\log_a(A/B) = \log_a\left(\frac{a^u}{a^v}\right) = \log_a(a^{u-v}) = u - v$$

$$= \log_a A - \log_a B$$

③ $\log_a(A^c) = c\log_a A$

$$\log_a(A^c) = \log_a((a^u)^c) = uc$$
$$= c(\log_a A)$$

IN YOUR homework you will be asked to either

① expand the expression:

$$\log_2(6x) = \log_2(6) + \log_2(x) \quad \underline{\text{property 5}}$$

② combining logarithms into single expression:

$$3\log x + \frac{1}{2}\log(x+1)$$

$$= \log x^3 + \log(x+1)^{1/2} \quad \underline{\text{property 7}}$$

$$= \log\left(x^3(x+1)^{1/2}\right) \quad \underline{\text{property 5}}$$

<u>expand examples:</u>

(10) $\log \frac{a^3}{3} = \log a^3 - \log 3$ property 6

$= 3\log a - \log 3$ property 7

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(20) $\log_2 \frac{32}{xy^2} = \log_2 32 - \log(xy^2)$ property 6

try to see if you can simplify → $\boxed{= \log_2 2^5} - \left(\log_2(x) + \log_2(y^2)\right)$

$= 5 - \log_2(x) - 2\log_2(y)$

Also: common mistakes

(1)

$\log(x+y)$ → cannot be expanded

$\log(x) + \log(y) = \log(xy)$

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(2)

$\log(x)\log(y)$ → cannot be expanded

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(3) $\log(x-y)$ cant be exanded

$\log(x) - \log(y) = \log\left(\frac{x}{y}\right)$

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ex.

$\log(x^2+y^2) = \log(x^2+y^2)$

$\log(x^2) + \log(y^2) = \log(x^2y^2)$

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(26) $\log\sqrt{\frac{xy^2}{z}} = \log\left(\frac{xy^2}{z}\right)^{1/2}$

$= 1/2 \log\left(\frac{xy^2}{z}\right)$

$= \frac{1}{2}\left[\log(xy^2) - \log(z)\right]$

$= \frac{1}{2}\left[\log(x) + \log(y^2) - \log(z)\right]$

$= \frac{1}{2}\left[\log(x) + 2\log(y) - \log(z)\right]$

$= \frac{1}{2}\log(x) + \log(y) - \frac{1}{2}\log(z)$

Write the expression as a logarithm of a single expression:

(32) $\log\sqrt{2} + \log\sqrt[3]{2}$

before you do any simplifying make sure all the terms have the same base

#5 $= \log\left((\sqrt{2})(\sqrt[3]{2})\right)$

$= \log\left((2^{1/2})(2^{1/3})\right)$ $\quad 2^{1/2+1/3} = 2^{5/6}$

$= \log(2^{5/6})$

$\boxed{= 5/6 \log(2)}$

ex.

$\log 33 - \ln 11$ $\rightarrow$ can't be simplified because they don't have same base

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(34)

$\ln(33) - \ln(11) = \ln\left(\frac{33}{11}\right) = \ln(3)$

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(38) $(\log_3 5 - \log_3 10) - \log_3 {}^1/_2$

$= \log_3\left(\frac{5}{10}\right) - \log_3({}^1/_2)$

$= \log_3({}^1/_2) - \log_3({}^1/_2)$

$= 0$

$\log_3 5 - [\log_3(10) + \log_3({}^1/_2)]$

$\log_3 5 - [\log_3(5)]$

$= 0$

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(44) $\log \sqrt[4]{x} + \log x^4 - \log x^2$

$(\log(x^{1/4}) + \log x^4) - \log x^2$

$\log(x^{1/4} x^4) - \log(x^2)$

$\log(x^{17/4}) - \log(x^2)$

$= \log\left(\frac{x^{17/4}}{x^{8/4}}\right)$

$= \log(x^{9/4})$

${}^9/_4 \log(x)$

(52) $\left(\log_3 x + \log_3 \sqrt{x+3}\right) - \frac{1}{3}\log_3 (x-4)$

$\log_3 (x\sqrt{x+3}) - \frac{1}{3}\log_3 (x-4)$

$\log_3 (x\sqrt{x+3}) - \log_3 (x-4)^{1/3}$

$\boxed{= \log_3 \frac{x\sqrt{x+3}}{(x-4)^{1/3}}}$

Calculator:

you have buttons for

log ln

$\log(500) = 2.69$ $\ln(5) = 1.609$

$\log(2) = 0.301$ $\ln(3) = 1.09$

But how do you calculate logarithms with different bases?

Change of base formula:

Suppose we are able to calculate $\underline{\log_a}$ (ln, log) but want to calc $\underline{\log_b x}$

$$y = \log_b x \Rightarrow b^y = x$$

$$\Rightarrow \log_a b^y = \log_a x$$

$$y(\log_a b) = \log_a x$$

$$y = \frac{\log_a x}{\log_a b}$$

$$\Rightarrow \boxed{\log_b x = \frac{\log_a x}{\log_a b}}$$

examples:

a) $\log_8 5 =$

change of base log

$$\log_8 5 = \frac{\log 5}{\log 8} = \frac{.69}{.903} \approx .76$$

Change of base with ln

$$\log_8 5 = \frac{\ln 5}{\ln 8} = \frac{1.609}{2.07} \approx .78$$