

# Key

## Math 113 Quiz 4

Find the derivatives of the following functions:

$$1) g(x) = \sqrt{x} \sin x = x^{1/2} \sin x$$

$$g'(x) = \frac{1}{2} x^{-1/2} \sin x + x^{1/2} \cos x$$

$$= \frac{1}{2\sqrt{x}} \sin x + \sqrt{x} \cos x$$

$$2) h(t) = \frac{t^2 + 2}{2t - 7}$$

$$h'(t) = \frac{2t + (2t - 7) - 2(t^2 + 2)}{(2t - 7)^2} = \frac{4t^2 - 14t - 2t^2 - 4}{(2t - 7)^2}$$

$$= \frac{2t^2 - 14t - 4}{(2t - 7)^2}$$

$$3) f(x) = e^{-x^2}$$

$$= (e^{-x^2})(-2x)$$

$$= \frac{-2x}{e^{x^2}}$$

$$4) p(x) = \sin(x^3)$$

$$= \cos(x^3) 3x^2$$

$$= 3x^2 \cos(x^3)$$

$$5) m(x) = \ln(x^2 + 1)$$

$$= \frac{1}{x^2 + 1} (2x) = \frac{2x}{x^2 + 1}$$

3 out of 5

8 points

a piece

6) Use the quotient rule to prove that  $\frac{d}{dx}[\cot x] = -\csc^2 x$

$$\begin{aligned}\frac{d}{dx}[\cot x] &= \frac{d}{dx}\left[\frac{\cos x}{\sin x}\right] = \frac{(-\sin x)(\sin x) - (\cos x)(\cos x)}{\sin^2 x} \\ &= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} \\ &= \frac{-1(\sin^2 x + \cos^2 x)}{\sin^2 x} \\ &= \frac{-1}{\sin^2 x} \\ &= -\csc^2 x\end{aligned}$$

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7) Find  $dy/dx$  by implicit differentiation for  $x^3y^3 - y = x$

$$\begin{aligned}\frac{d}{dx}[x^3y^3 - y] &= \frac{d}{dx}[x] \\ &= \frac{d}{dx}[x^3y^3] - \frac{d}{dx}[y] = 1 \\ &= 3x^2y^3 + 3y^2\frac{dy}{dx}x^3 - \frac{dy}{dx} = 1\end{aligned}$$

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$$3y^2\frac{dy}{dx}x^3 - \frac{dy}{dx} = 1 - 3x^2y^3$$

$$\frac{dy}{dx}(3y^2x^3 - 1) = 1 - 3x^2y^3$$

$$\boxed{\frac{dy}{dx} = \frac{1 - 3x^2y^3}{3y^2x^3 - 1}}$$