

Related Rate Problem

- 1) The radius r of a circle is increasing at a rate of 5 centimeters per minute. Find the rate of change of the area when the radius is 10 cm.

$$A = \pi r^2$$

$$\frac{dr}{dt} = 5 \text{ cm/minute} \quad r = 10$$

$$\frac{d}{dt} [A] = \frac{d}{dt} [\pi r^2]$$

$$\Rightarrow \frac{dA}{dt} = 2\pi r \frac{dr}{dt} \quad \Rightarrow \quad \frac{dA}{dt} = 2\pi (10 \text{ cm}) (5 \text{ cm/minute})$$

$$= 314.16 \text{ cm}^2/\text{minute}$$

Rolle's Theorem

- 2) Determine whether Rolle's Theorem can be applied to $f(x) = (x-2)(x+1)$ on the closed interval $[-1, 2]$. If Rolle's theorem can be applied then find all values of c such that $f'(c) = 0$

For Rolle's theorem

$$f(-1) = f(2)$$

$$f(-1) = (-1-2)(-1+1) = 0$$

$$f(2) = (2-2)(2+1) = 0$$

then there exist a number c , $-1 \leq c \leq 2$ such that

$$f'(c) = 0$$

$$f(x) = x^2 + x - 2x - 2 \Rightarrow f(x) = x^2 - x - 2$$

$$\Rightarrow f'(x) = 2x - 1 \quad \text{set } f'(x) = 0 \quad \Rightarrow \quad \boxed{x = 1/2}$$

$$\text{and } -1 < 1/2 < 2$$

Critical Points

3) Find the critical numbers for the function $f(x) = \cos(x)$ on the interval $[0, 2\pi]$

$$f(x) = \cos x$$

critical numbers when $f'(x) = 0$ or $f'(x)$ is undefined.

$$f'(x) = -\sin x$$

$$0 = -\sin x \Rightarrow f'(x) = -\sin x = 0$$

$$x = 0, \pi, 2\pi$$

So critical points at

$$(0, f(0)) = (0, 1), (\pi, f(\pi)) = (\pi, -1), (2\pi, f(2\pi)) = (2\pi, 1)$$

Relative Extrema

4) Find all relative extrema. Use the second derivative test to show whether the extrema are relative minimum or maximums.

$$f(x) = x^3 - 3x^2 + 3$$

Relative extrema exist when $f'(x) = 0$ or $f'(x)$ is undefined.

$$f'(x) = 3x^2 - 6x$$

$$f'(x) = 3x^2 - 6x = 0$$

$$3x(x-2) = 0$$

$$3x = 0 \text{ or } x = 2$$

$$f''(x) = 6x - 6$$

$$f''(0) = 6(0) - 6 = -6$$

Then $(0, 3)$ is a relative Maximum

$$f''(2) = 6(2) - 6$$

Then $(2, -1)$ is a relative Maximum.