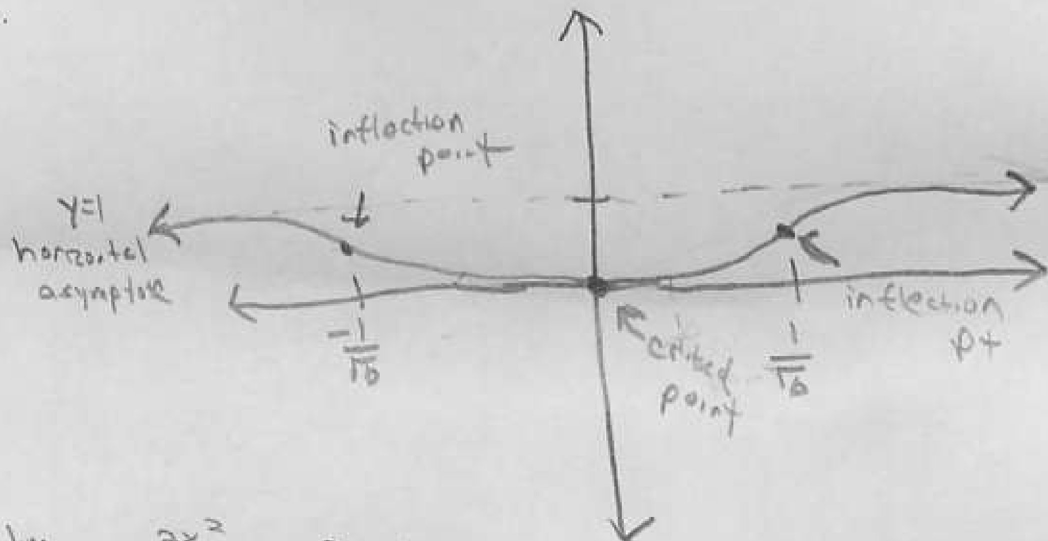
Key

Math 113
Quiz 6

1) Analyze and sketch a graph of the function $f(x)=\dfrac{2x^2}{2x^2+1}$



① zeros: (0,0)

$2x^2=0$

$x=0$

② vertical asymptotes

$2x^2+1=0$

complex so no V. A

③ Horizontal asymptote

$\lim_{x\to\infty}\dfrac{2x^2}{2x^2+1}=\frac{2}{2}=1=y$ $\quad\lim_{x\to-\infty}\dfrac{2x^2}{2x^2+1}=\frac{-2}{-2}=1=y$

④ critical points: $f'(x)=0$ or $f'(x)$ is undefined

$$f'(x)=\frac{4x(2x^2+1)-2x^2(4x)}{(2x^2+1)^2}=\frac{8x^3+4x-8x^3}{(2x^2+1)^2}=\frac{4x}{(2x^2+1)^2}$$

$f'(x)=0$ when $4x=0 \Rightarrow x=0$ critical point (0,0)

⑤ possible inflection pts. $f''(x)=0$ or $f''(x)$ is undefined

$$f''(x)=\frac{4(2x^2+1)^2-4x(2(2x^2+1))(4x)}{(2x^2+1)^4}=\frac{(2x^2+1)\left[4(2x^2+1)-16x^2(2)\right]}{(2x^2+1)^4}$$

$$=\frac{8x^2+4-32x^2}{(2x^2+1)^3}=\frac{-24x^2+4}{(2x^2+1)^3}$$

$f''(x)=0$ when $-24x^2+4=0$

$x^2=\frac{1}{6}$

$x=\pm\sqrt{1/6}$

Tables:

inc/dec

	-1	1
$f'(x)$	−	+

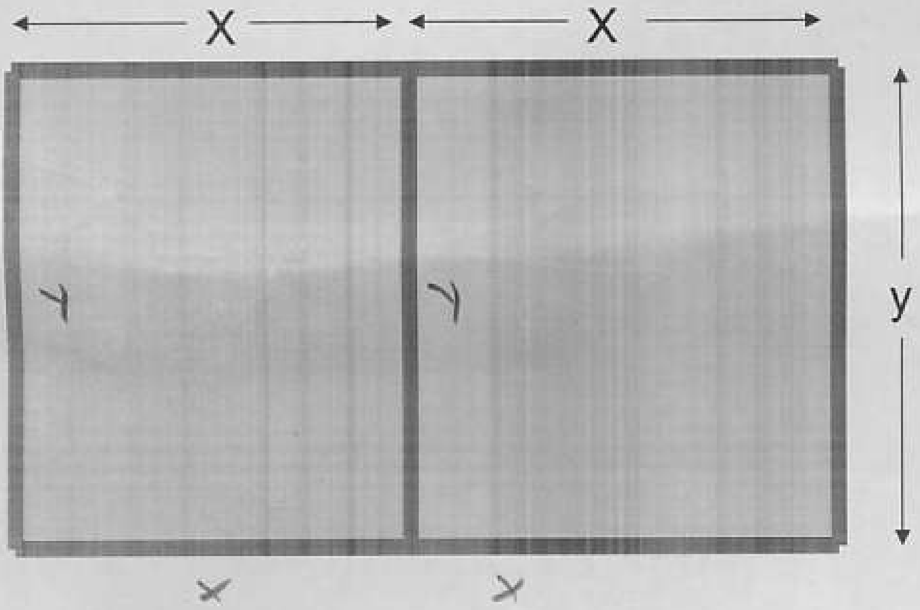
increasing $(0,\infty)$ decreasing $(-\infty,0)$

concave

	-1	0	1
$f''(x)$	−	+	−

concave down $(-\infty,-1/\sqrt{6})$ $(1/\sqrt{6},\infty)$ c.U $(-1/\sqrt{6},1/\sqrt{6})$

2) A rancher has 200 feet of fencing with which to close two adjacent rectangular corrals What dimensions should be used so that the enclosed area will be maximum?



$4x + 3y = 200$

primary $A = 2(xy)$ ① goal maximize A.

secondary ~~maximize~~ $4x + 3y = 200$ ft

$4x = 200 - 3y$

$x = 50 - \frac{3}{4}y$ ②

Substitute 2 into ①

$A = 2(50 - \frac{3}{4}y)(y)$

$A = 100y - \frac{3}{2}y^2$

$\frac{dA}{dy} = 100 - 3y = 0$

$-3y = -100$

$\boxed{y = 33\frac{1}{3} \text{ feet}}$

$y = 33\frac{1}{3}$ feet

$x = 50 - \frac{3(33\frac{1}{3})}{4}$

$x = 50 - 25$

$x = 25$ feet

Maximum: when

$x = 25$ feet

$y = 33\frac{1}{3}$ feet

check boundaries

① $y = 0 \Rightarrow A = 0$

② $x = 0 \Rightarrow A = 0$

so max