

Key

Math 113  
 Exam #1  
 (Chapter 2-Limits)  
 Due: Beginning of class July 18, 2007

I pledge all work on this exam was done by me without the assistance of my classmates or any other individual. I realize that by cheating I am wasting both my time and money.

Signature \_\_\_\_\_

1) Complete the tables and use your result to estimate the limit.

a)  $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

x	0.1	0.001	0.0001	-0.0001	-0.001	-0.1
f(x)	~.998	.999	~1	~1	~.999	~.998

if x was in radians:

if x was in degrees

	.1	.001	.0001	-.0001	-.001	-.01
	.0175	.0175	.0175	.0175	.0175	.0175

$\lim_{x \rightarrow 0} \frac{\sin x}{x} = .0175$

b)  $\lim_{x \rightarrow 0} (1+x)^{1/x} = e$

x	0.1	0.001	0.0001	-0.0001	-0.001	-0.1
f(x)	~2.593	2.716	2.718	2.718	2.719	2.86

What is the mathematical symbol for this value?

"e"

2) Find the limits:

a)  $\lim_{\Delta x \rightarrow 0} \frac{\sqrt{x+\Delta x} - \sqrt{x}}{\Delta x}$

$= \lim_{\Delta x \rightarrow 0} \frac{\sqrt{x+\Delta x} - \sqrt{x}}{\Delta x} \left( \frac{\sqrt{x+\Delta x} + \sqrt{x}}{\sqrt{x+\Delta x} + \sqrt{x}} \right)$  foil

$= \lim_{\Delta x \rightarrow 0} \frac{(x+\Delta x) - x}{\Delta x (\sqrt{x+\Delta x} + \sqrt{x})}$

$= \lim_{\Delta x \rightarrow 0} \frac{\Delta x}{\Delta x (\sqrt{x+\Delta x} + \sqrt{x})}$

$= \lim_{\Delta x \rightarrow 0} \frac{1}{\sqrt{x+\Delta x} + \sqrt{x}}$  ← Direct substitution

$= \frac{1}{2\sqrt{x}}$

b)  $\lim_{x \rightarrow 0} \frac{1 - \cos^2 x}{x}$  (Hint: Use your result from 1a)

$\lim_{x \rightarrow 0} \frac{1 - \cos^2 x}{x} = \lim_{x \rightarrow 0} \frac{\sin^2 x}{x}$

$= \lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot \frac{\sin x}{1}$

$= \lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot \lim_{x \rightarrow 0} \sin x$

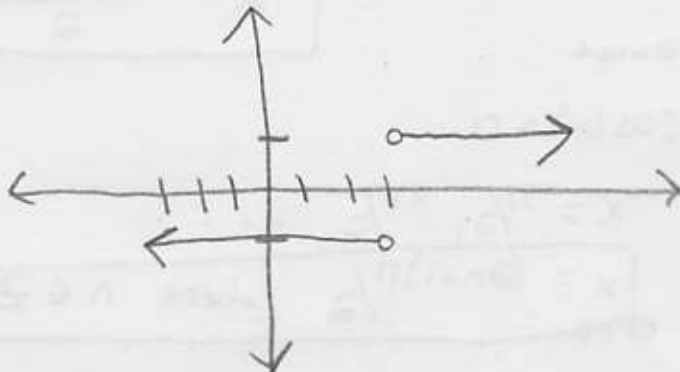
$= 1 \cdot 0$

$= 0$

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3) For  $f(x) = \frac{|x-3|}{x-3}$ :

a) Draw a graph of  $f(x)$ :



b) Use a to find  $\lim_{x \rightarrow 3^+} f(x) = +1$

c) Use a to find  $\lim_{x \rightarrow 3^-} f(x) = -1$

d) Use your results from b and c to determine  $\lim_{x \rightarrow 3} f(x) = \text{DNE}$

e) Is this function continuous? Use the definition of continuity (pg. 90) to justify your answer?

This function is not continuous because  $\lim_{x \rightarrow 3} f(x)$  does not exist.

4) Find all values of  $x$  for which the function  $f(x) = \sec(x)$  is not continuous.

$f(x) = \sec x$   
 $f(x) = \frac{1}{\cos(x)}$

So  $f(x)$  is not continuous when  
 $x = \frac{(2n+1)\pi}{2}$  where  $n \in \mathbb{Z}$

Asymptotes exist

$$\cos(x) = 0$$

$$\Rightarrow x = \pi/2, 3\pi/2, \dots$$

$$x = \frac{(2n+1)\pi}{2} \text{ where } n \in \mathbb{Z}$$

5) Let  $f(x) = x^2 - 6x + 8$ .

a) Use the intermediate value theorem to prove that this function crosses the  $x$ -axis on the interval  $0 \leq x \leq 3$

Since  $f(x)$  is a continuous polynomial  
To prove use intermediate value theorem but  
show that  $f(0)$  and  $f(3)$  have different signs

$$f(0) = 0^2 - 6(0) + 8 = 8 = \text{positive}$$

$$f(3) = 3^2 - 6(3) + 8 = -1 = \text{negative}$$

Therefore by IVT there exist a  $c$  such that  $f(c) = 0$

b) Find the zero of the function guaranteed by the theorem.

To find the  $c$  guaranteed by (a) solve

$$x^2 - 6x + 8 = 0$$

$$(x-4)(x-2) = 0$$

$$x = 4 \text{ or } x = 2$$

$x = 2$  is in the interval  $[0, 3]$

6) Let  $f(x) = \frac{x-3}{x^2-9}$

a) Find the domain of  $f(x)$ :

1/10 Domain of  $f(x)$  is restricted when  $x^2-9=0$  so domain  $f(x) = \{x \mid x \neq 3, -3\}$

b) Find the values of which  $f(x)$  is not continuous? Which of these values are removable?

1/5  $f(x)$  is not continuous at  $x=3, x=-3$  because  $\frac{x-3}{(x+3)(x-3)} = \frac{1}{x+3}$  removable discontinuity at  $x=3$

c) Find any asymptotes to this function?

1/5 Asymptote exist at  $x=-3$

d) Use your answer from part c to determine the limit of  $f(x)$  as it approaches the asymptote from the left hand side and the right hand side?

1/10  $\lim_{x \rightarrow -3^+} f(x) = +\infty$

$\lim_{x \rightarrow -3^-} f(x) = -\infty$

	$-3^+$	$-3^-$
$x-3$	+	-
$(x+3)(x-3)$	+	-
$y$	$+\infty$	$-\infty$