Math 113
Exam #2
(Chapter 3-Derivatives)
Due: Beginning of class July 30,2007

1) Let $f(x) = \frac{5}{\sqrt{x}}$

a) Find the derivative of f(x) using the limit process.

$$f'(x) = \lim_{\Delta x \to 0} \frac{\frac{5}{\sqrt{x+\Delta x}} - \frac{5}{\sqrt{x}}}{\Delta x} = \lim_{\Delta x \to 0} \frac{5\sqrt{x} - 5\sqrt{x+\Delta x}}{\sqrt{x}\sqrt{x+\Delta x}\,\Delta x} \frac{(5\sqrt{x} + 5\sqrt{x+\Delta x})}{(5\sqrt{x} + 5\sqrt{x+\Delta x})}$$

$$= \lim_{\Delta x \to 0} \frac{25x - 25(x+\Delta x)}{\sqrt{x}\sqrt{x+\Delta x}\,\Delta x\,(5\sqrt{x}+5\sqrt{x+\Delta x})}$$

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$$= \lim_{\Delta x \to 0} \frac{-25\Delta x}{\Delta x\,\sqrt{x}\sqrt{x+\Delta x}\,(5\sqrt{x}+5\sqrt{x+\Delta x})}$$

$$= \lim_{\Delta x \to 0} \frac{-25}{\sqrt{x}\sqrt{x+\Delta x}(5\sqrt{x}+5\sqrt{x+\Delta x})} = \frac{-25}{(\sqrt{x})(\sqrt{x})(10\sqrt{x})}$$

$$= \frac{-5}{2\sqrt[2]{x^3}} = \boxed{\left(\frac{-5}{2}\right)x^{-3/2}}$$

b) Find the derivative of f(x) using the basic differentiation rules (Make sure this is the same as part a)

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$$f(x) = \frac{5}{\sqrt{x}} = 5(x)^{-1/2} \Rightarrow f'(x) = \frac{-1}{2}5x^{-1/2-2/2} \boxed{= \frac{-5}{2}x^{-3/2}}$$

c) Evaluate the slope of the tangent line to f(x) at the point (4,5/2)

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The slope of the tangent line

$$f'(4) = \frac{-5}{2}(4)^{-3/2} \boxed{= -\frac{5}{16}}$$

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2) An airplane drops humanitarian aid. When pushed from the plane, the altitude of the package is given by $h(t) = -16t^2 - t + 30{,}000$ (feet)

a. What is the instantaneous velocity of the package after 10 seconds?

instenecus velocty at 10 seconds

~~find~~ $h'(10) =$

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$h'(t) = -32t - 1$

$h'(10) = -320 - 1 = -321$ ft/second

b. How long does it take the package to reach earth?

$-16t^2 - t + 30000 = 0$

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$x = \dfrac{-(-1) \pm \sqrt{(1)^2 - 4(-16)(30000)}}{-32} \approx 43.3$ seconds

c. What is instantaneous velocity when the object hits the ground?

$h'(43.3) = -32(43.3) - 1 = -1386.6$ ft/second.

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d. What is the packages acceleration when it hits the ground?

$h''(t) = -32$ ft/sec^2 ← acceleration due to gravity.

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$h''(43.3) = -32$ ft/sec^2

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3) For the function $g(x) = \arctan(x)$:

a) Find an equation for the tangent line to $g(x)$ at the point [redacted] $(-1, -\pi/4)$

$g(x) = \arctan(x)$

$g'(x) = \dfrac{1}{1+x^2}$

/10 $\boxed{g'(-1) = \dfrac{1}{1+(-1)^2} = \dfrac{1}{2}}$

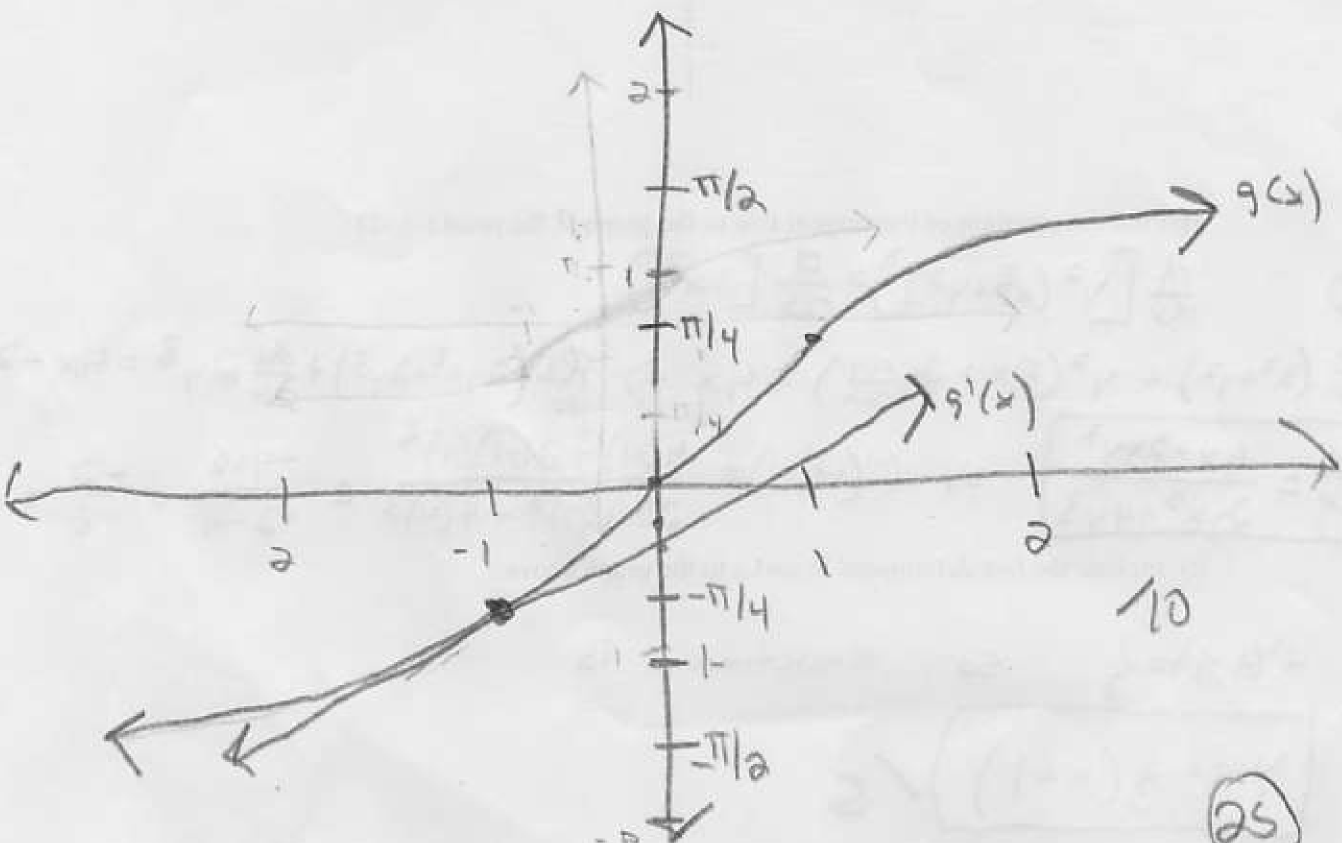
$y - y_1 = m(x - x_1)$

$y + \pi/4 = \frac{1}{2}(x+1)$

$y = \frac{1}{2}x + \frac{1}{2} - \pi/4$

$\boxed{y = \frac{1}{2}x + \frac{2-\pi}{4}}$ /5 $\boxed{y = \frac{1}{2}x - .285}$

b) Sketch a graph of $g(x)$ and the line that you found in part a.



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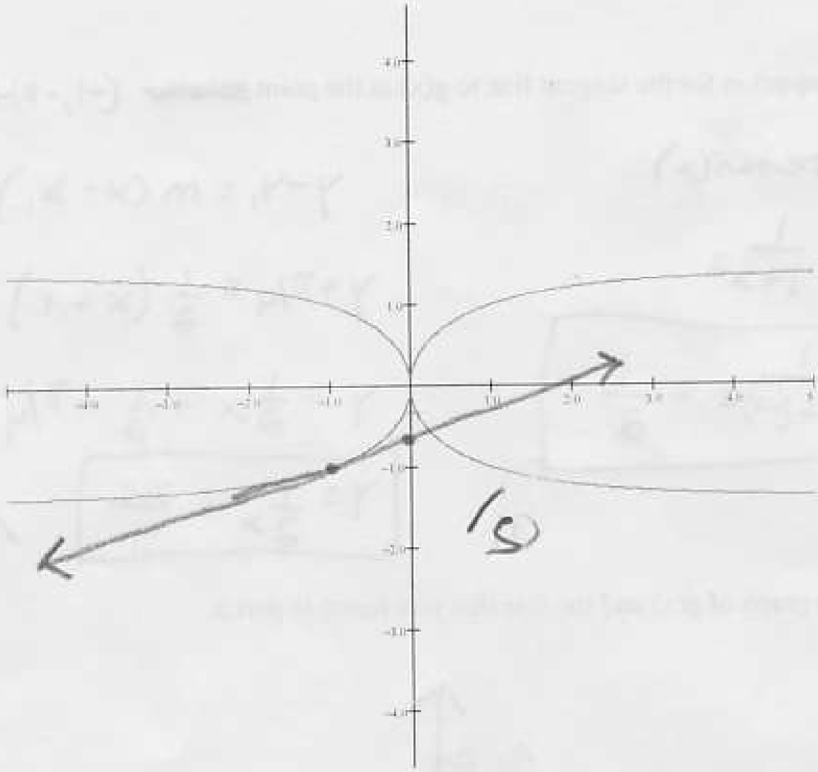
4) Consider the kappa curve: $y^2(x^2+y^2)=2x^2$



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a) Find the equation of the tangent line to the graph at the point (-1,-1)

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$$\frac{d}{dx}\left[y^2(x^2+y^2)\right]=\frac{d}{dx}\left[2x^2\right]$$

$$\Rightarrow 2y\frac{dy}{dx}(x^2+y^2)+y^2\left(2x+2y\frac{dy}{dx}\right)=4x \Rightarrow \frac{dy}{dx}(2yx^2+2y^3)+\frac{dy}{dx}2y^3=4x-2xy^2$$

$$\Rightarrow \boxed{\frac{dy}{dx}=\frac{4x-2xy^2}{2yx^2+4y^3}} \Rightarrow f'(-1,-1)=\frac{4(-1)-2(-1)(-1)^2}{2(-1)(-1)^2+4(-1)^3}=\frac{-4+2}{-2-4}=\frac{-2}{-6}=\frac{1}{3}$$

b) Include the line determined in part a to the graph above.

Then $f'(-1,-1)=\frac{1}{3}$ so equation is

$\boxed{y+1=\frac{1}{3}(x+1)}$ /5

$$y=\frac{1}{3}x+\frac{1}{3}-1 \Rightarrow y=\frac{1}{3}x-\frac{2}{3}$$

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5) Find $\frac{dy}{dx}$ using logarithmic differentiation where $y = \frac{(x+1)(x+2)}{(x-1)(x-2)}$

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$$\ln y = \ln\left(\frac{(x+1)(x+2)}{(x-1)(x-2)}\right)$$

$$\ln y = \ln(x+1) + \ln(x+2) - \ln(x-1) - \ln(x-2)$$

$$\frac{d}{dx}[\ln y] = \frac{d}{dx}[\ln(x+1)] + \frac{d}{dx}[\ln(x+2)] - \frac{d}{dx}[\ln(x-1)] - \frac{d}{dx}[\ln(x-2)]$$

$$\frac{dy}{dx}\frac{1}{y} = \frac{1}{x+1} + \frac{1}{x+2} - \frac{1}{x-1} - \frac{1}{x-2}$$

$$\frac{dy}{dx} = y\left(\frac{1}{x+1} + \frac{1}{x+2} - \frac{1}{x-1} - \frac{1}{x-2}\right) \Rightarrow \frac{dy}{dx} = \left[\frac{(x+1)(x+2)}{(x-1)(x-2)}\right]\left(\frac{1}{x+1} + \frac{1}{x+2} - \frac{1}{x-1} - \frac{1}{x-2}\right)$$

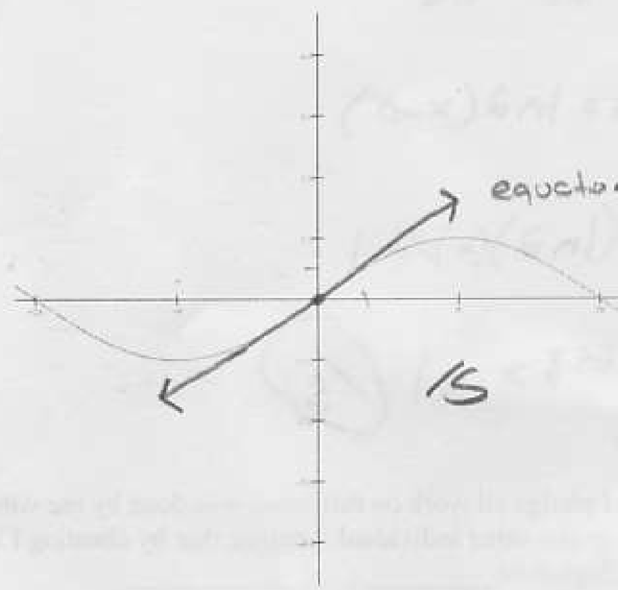
6) Find the slope of the tangent line to f(x)=sin(x/2) at the origin. Include it in the graph below.

$f(x) = \sin(x/2)$

$f'(x) = \cos(x/2)(1/2)$

$f'(0) = \cos(0)(1/2)$ /5

$= 1/2$



equation of tangent at origine

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equation of tangent line:

$y - 0 = 1/2(x-0)$

$y = 1/2x$ /5

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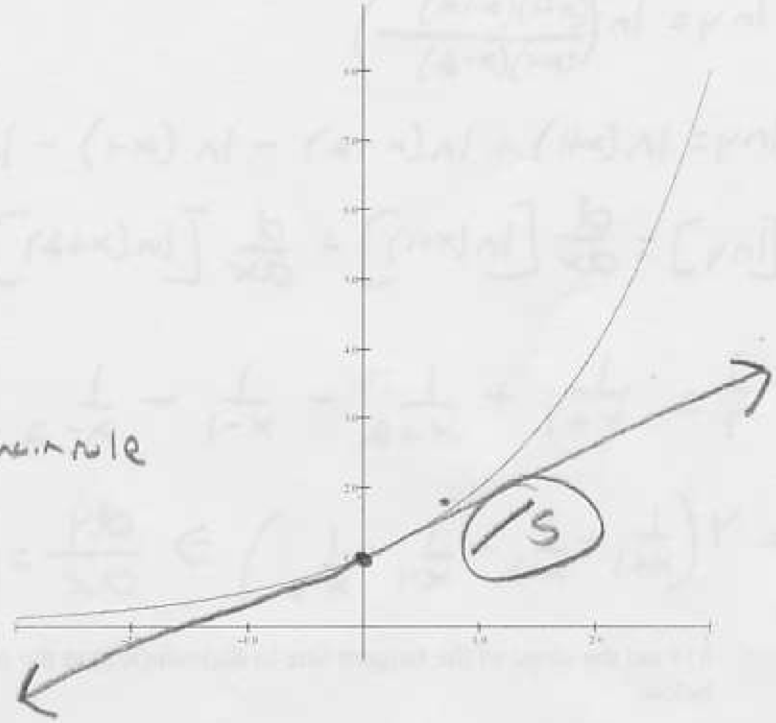
7) Find the slope of the tangent line to $f(x) = 2^x$. at the point (0,1) Include it in the graph below.



$f(x) = e^{\ln 2^x}$

$f(x) = e^{x \ln 2}$

$\Rightarrow f'(x) = e^{x\ln 2}(\ln 2)$ using chain rule

$f'(x) = (2^x)(\ln 2)$

$f'(0) = (2^0)(\ln 2)$

$f'(0) \approx .693$ /15

equation of line

$y - 1 = \ln 2(x - 0)$

$y = (\ln 2)(x) + 1$

$y \approx .693x + 1$ /5

I pledge all work on this exam was done by me without the assistance of my classmates or any other individual. I realize that by cheating I am wasting both my time and money.
Signature______________________________

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